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# PROCEEDINGS

OF THE

# AMERICAN ACADEMY

OF

## ARTS AND SCIENCES.

VOL. XXI.

PAPERS READ BEFORE THE ACADEMY.

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INVESTIGATIONS ON LIGHT AND HEAT, MADE AND PUBLISHED WHOLLY OR IN PART WITH  
APPROPRIATION FROM THE RUMFORD FUND.

### I.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

## XX.—ON THE EFFECT OF TEMPERATURE ON THE VISCOSITY OF AIR.

BY SILAS W. HOLMAN.

Communicated May 13, 1885.

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THE investigation described in the present paper is a continuation of the work done on the same subject by the author in 1876.\* The experimental portion of the work consists of about 250 independent measurements of the viscosity of air and 140 of carbon dioxide (carbonic acid), besides somewhat extended incidental researches on thermometry. The temperature ranges for air were from 0° to 124° C.; for carbonic acid, from 0° to 225° C. The plan adopted at the outset was to make some study of the method and of the best form of apparatus; then to proceed to the study of several gases which have been

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\* On the Effect of Temperature on the Viscosity of Air, Proc. Amer. Acad. Arts and Sci., xii. (1876), p. 41. Phil. Mag., iii. 81 (1876). Wied. Beibl., i. 222.

already well investigated, either in regard to their viscosity or other properties, making measurements of the viscosity at temperature intervals of about  $20^{\circ}$  from low to high temperatures. But the laborious nature of the observations, and the large amount of time demanded by the observations and their reduction, have rendered the fulfilment of the plan impossible for one whose available time has been fully occupied with laboratory instruction. It will be seen that the last of the observations given were made in April, 1880. Since that date it has been wholly inconsistent with the author's duties and health to continue the work, and the incomplete results are now presented in the hope that, despite their limited range, they may be a contribution of some permanent value to the subject treated.

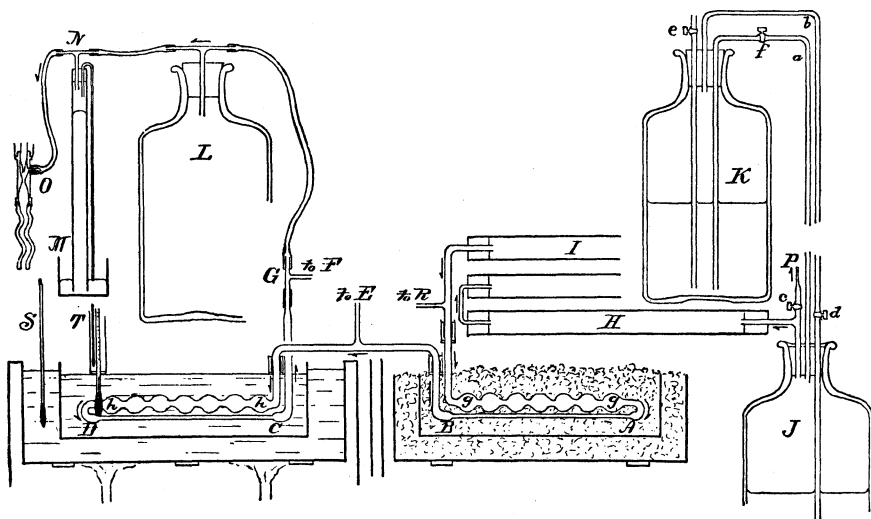
The expenses of the investigation have been mainly defrayed by an appropriation granted by the American Academy of Arts and Sciences from the Rumford Fund for researches on Light and Heat.

The objects of the investigation were concisely stated in the paper of 1876. The extension of the work was looked to for the development of data which not only should furnish another experimental check on the deductions of the kinetic theory of gases, but should provide material of sufficient precision (approximating to that with which the coefficient of expansion of gases is known) to serve as a part of that experimental basis on which the complete theory of gases, and indeed of molecular physics in general, must eventually rest. The critical review of the method, given at the end of the paper, is intended to furnish the material for the formation of a judgment as to how far this object has been fulfilled in the case of the gases investigated.

#### APPARATUS.

The apparatus used is shown in the schematic section given by the accompanying cut. It is in principle the same as that used in the former measurements. The dried gas is transpired successively through two glass capillaries of about 30 cm. in length, from a space containing the gas at a measured pressure to another space from which the gas is continuously exhausted as fast as it enters, and which is thus maintained at a constant and measured pressure. The pressure of the gas in the intermediate space is also measured. The capillaries are maintained at known or measured constant temperatures, *AB* being usually packed in finely pounded ice, and *DC* being in a jacketed double oil-bath, of which the temperature is maintained nearly constant by regulating the flame of the gas-burners, and is measured by the thermometer with its

bulb at *D*, the entrance to the capillary. Both inner and outer oil baths are continuously stirred by an agitator moved to and fro through the troughs by a water motor with connections not shown in the sketch. This arrangement maintained temperatures as high as  $225^{\circ}$  constant within about  $0^{\circ}.5$  in most measurements, and so nearly uniform throughout the trough that no perceptible difference could be found at various points along the capillary. A thermometer at *S* in the outer bath served to adjust the lamps so that their heating should be such as to maintain a nearly constant temperature. Automatic thermo-regulators were tried, but were found in general to be quite as much of a disadvantage as benefit, and were abandoned.



The details of the apparatus will be described, beginning at the point where the gas enters. When air was used, the laboratory air was drawn in through caustic potash to remove the carbonic acid, and through concentrated sulphuric acid, fused chloride of calcium, and anhydrous phosphoric acid in the tubes *H* and *I*, which are further described at I. 2, p. 29. When carbonic acid was used, the gas was formed by the action of hydrochloric acid on marble, was purified by washing in water, dried, and freed from hydrochloric acid by passing through tubes of pumice coated with anhydrous copper sulphate. The gas was then drawn through *P* into *J* by withdrawing the concentrated sulphuric acid from *J* into *K*, as about to be described, and was there held for use. When a measurement was in progress, the gas in *J*, being

slowly and uniformly displaced by the sulphuric acid returning from *K*, passed into and through *H* and *I*, arranged as already described. The glass bottles *J* and *K* held each about 10 litres, and one was filled with concentrated sulphuric acid. *K* was placed about a meter above *J*. The two were connected by two glass tubes, *a* and *b*, each containing a stop-cock, and had other tubes arranged as shown in the sketch. To withdraw the sulphuric acid from *J* into *K*, and to replace this by the  $\text{CO}_2$ , it was necessary simply to connect *P* to the gas-holder or generator, to close *e* and *f*, to open *c*, to exhaust the air from *K* by an aspirator, and, opening the cock in *b*, to allow the  $\text{H}_2\text{SO}_4$  to be transferred to any desired extent. This being accomplished, *d* and *c* were closed and the generator disconnected. When a measurement is in progress, the  $\text{CO}_2$  passing at a uniform rate from *J* through *Q* must be replaced by sulphuric acid from *K* through *a*. This flow is regulated by a tangent screw motion on the cock *f*, until the liquid drops or runs into *J* at such a rate as to displace not only the necessary amount of  $\text{CO}_2$ , but slightly more than this, the excess escaping slowly through *c* and a fine orifice at *P*. This arrangement maintains an almost constant pressure in *J*, and, as the pressure is thus always outward, there can be no inward leakage of air either during the measurements or at other times. The gas, after passing, always with this slight excess of pressure within, through *HI*, enters the glass bulbs *g g*, and the pressure  $p_1$  at entrance is measured by the barometric height at the time plus the pressure indicated by the gauge at *R*, read by the cathetometer. The bulbs *g g* were spherical enlargements of about three centimeters in diameter of a glass tube of about half a centimeter in diameter. The contents of these bulbs would be transpired by the capillaries once in about fifteen minutes. The tube *g g* and the capillary were connected into one piece at *A* by melting them together, and a similar solid connection was made at *B* to an exit tube. The tubes *CD* and *h h* were similar in all respects to *AB* and *g g*, the capillaries being of very nearly the same length, and cut from the same piece of tubing, which was one of those used in the experiments of 1876. The connections of these tubes to the other tubing of the apparatus, as shown in the drawing, were all made as described in I. 3, p. 29. The tube connecting *B* to *h h* was of about 3 mm. internal diameter and 50 cm. length, and at its middle point contained a branch, which extended, with one joint of the kind described, to the gauge *E*, which in connection with the barometer served to measure  $p_2$ . The gas at exit from *C* passed about 50 cm. of 3 mm. tubing to the point *G*, where a side branch led to the gauge *F*, which gave the pressure  $p_3$ . Passing by *G*,

the gas moved forward past *N* to the aspirator *O*,\* and then escaped. The bottle *L*, of about 10 litres' capacity, sometimes two bottles each of that volume, connected between *G* and *N*, served to reduce the rate of pressure fluctuations arising from irregularities in the aspiration. At *N* was connected the pressure-regulator, which I will describe in detail, as it is so far as I know wholly new and is available for many purposes.

The aspirator under a given head of water can exhaust at a definite rate, maintaining a certain exhaustion; if the head be increased or lessened, the rate of exhaustion will be increased or diminished, and thus if the leakage into the exhausted space, as through the capillaries in this case, be sensibly constant, the pressure in the exhausted space must diminish or increase. It is often impracticable, as it was for me, to maintain a constant head of water, and some means is therefore necessary to overcome this irregular action. The arrangement used was that shown in the sketch. A large glass tube *M*, open at both ends, stands upright in an open mercury trough of considerably larger dimensions. Into its top is inserted a rubber stopper with two borings, through one of which passes one arm of a **T** joint whose other two arms are connected respectively with the aspirator and the vessel to be exhausted. Through the other boring passes a tube of 1 mm. to 2 mm. diameter bent twice at right angles, and dipping into the mercury in the trough. Suppose that the vessel to be exhausted is closed — the apparatus covers cases ranging from this to a leakage of about half the rate of exhaustion of which the pump is capable — and the aspirator set in operation. As the exhaustion proceeds, the mercury rises equally in both large and small tubes until the mercury in the trough drops below the point of the latter, whereupon, as soon as the excess of external pressure is sufficiently great to overcome the friction of the mercury, the column in the fine tube rises rapidly and flows over into the upper part of the large tube and upon the mercury surface in it. This overflow is of course immediately followed by an inrush of air through the open point, and a consequent sudden lowering of the mercury in the large tube with a corresponding rise of level in the trough which closes or partly closes *D*. This first sudden action is followed by several of lessening violence, and a steady condition is soon reached, in which there is a continuous inflow of air and small drops of mercury at the point, the

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\*Richard's jet aspirator was used. See Amer. Jour. Sci., [3], viii. 200; Chem. News, xxxiv. 141; Trans. Amer. Inst. Mining Eng., vi. 492 (1879).

proportion between the two depending upon the relation between the sizes of the tubes, the capacity of the aspirator, and the supply of gas from the vessel to be exhausted. If a fluctuation occurs in either the rate of the aspiration or the supply of gas to be exhausted, a greater or less proportion of air will be taken in, with a corresponding but very small change of the pressure within the apparatus. For the successful working of the apparatus a proper proportioning of parts to the work to be done is of course necessary, and this is mainly to be accomplished by varying the size of the point to which the fine tube is drawn out. The sensitiveness of the apparatus may be increased by using a **U** tube, instead of a vertical straight tube with cistern, and allowing the fine tube to extend into the open arm of this tube. A stricture at some point of the **U** reduces the too sudden fluctuations. Various other modifications will readily suggest themselves to persons using the apparatus. The regulator is of course of service in other ways than merely rendering uniform the action of the aspirator. It is possible to maintain with it a constant exhaustion of any desired amount up to the full exhaustion which the aspirator can produce when working under the smallest head of water likely to occur; and in the form of a **U** tube, with various lengths of tube *DE*, to be inserted at *A* and raised or lowered, it affords a very convenient means of varying the exhaustion at will.

#### PROCEDURE IN MEASUREMENTS.

The first operation is thoroughly to dry the apparatus and fill it with pure dry gas. This was accomplished by repeated exhaustion and refilling of all necessary parts of the tubing, gauges, etc., none but thoroughly dry gas being admitted. This operation was repeated on each separate day of measurement so that any error from slight leakage or diffusion of either air or moisture should be avoided. On the first introduction of the dry air or carbonic acid, the further precaution was taken to run the apparatus under quite high exhaustion for many hours consecutively; and in all cases, on each day of measurement, the apparatus was run for an hour or more before any readings were taken.

The procedure in measurement was as follows. The aspirator was set in operation, *J* being full of the gas. The generator and purifiers were connected at *P*, and the cock *c* opened, *f* remaining closed, so that the supply of gas came from the generator; the temperature of *DO* was regulated, the stirrer being in continuous operation; the ice-bath

was placed around  $AB$ , the regulator at  $N$  adjusted, and the transpiration allowed to proceed until the gauges  $E$  and  $F$  were nearly stationary. Then  $c$  was closed, and  $f$  opened and adjusted so that the reading of  $R$  slowly increased, whereupon  $c$  was again opened for the slow escape of the gas and the maintenance of a constant initial pressure  $p_1$ . The transpiration was allowed to proceed until  $E$ ,  $F$ , and  $R$  were sufficiently near stationary, when readings of the mercury columns by the cathetometer and of the thermometers were taken systematically. After a complete set of readings had been thus obtained, the apparatus was thoroughly inspected and a second set made; and so on until the number was deemed sufficient, whereupon the temperature of  $DC$  was changed or the pressure modified as desired. The results thus obtained furnished, on reduction, the values of  $p_1$ ,  $p_2$ ,  $p_3$ , and  $t$  for the computation of the ratio of the coefficient of viscosity at the higher temperature to that at  $0^\circ \text{C.}$ , the further requirements being the constants  $\frac{R_2^4 \lambda_1}{R_1^4 \lambda_2} = K$  for the tubes,  $R_1$  and  $\lambda_1$ ,  $R_2$  and  $\lambda_2$ , being the radius and length respectively of the tubes  $AB$  and  $CD$ , as shown at page 9. This value of  $K$  was found from measurements similar in all respects to those just described, except that the tubes  $AB$  and  $CD$  were both surrounded with ice at the same time.

The apparatus was also arranged to place  $DC$  in steam. When left at night for subsequent use, an outward pressure was created in all parts of the apparatus.

### EXPERIMENTAL RESULTS.

The results will be here given as they were for convenience classified during the progress of the work, viz. with dry air in five series, with carbonic acid in two series. The method of reduction of the results is given at page 26, and in my first paper.

#### *Air.*

*First Series.*—This consists of the twenty-one measurements made in 1876 as given at page 49 of the article referred to, and gives as a mean result of the ten measurements of  $\frac{\eta_1}{\eta_2}$  as shown in Table I. below, the value  $y = \frac{\eta_1}{\eta_2} = 1.234$ , with an average deviation\* of 0.044,

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\* Let  $a_1, a_2, a_3, \dots a_n$ , be a series of measurements of the same quantity, and let  $m$  be their arithmetical mean; then  $a_1 - m = d_1$ ,  $a_2 - m = d_2$ ,  $a_3 - m = d_3$ , &c., will be the deviation of these measurements from their



or about four per cent. A glance at the page referred to will show, however, that of these ten measurements but six were at the temperatures of  $0^\circ$  and  $100^\circ$  respectively. These six as shown in Table II. give the mean

$$y = \frac{\eta_{100}}{\eta_0} = 1.270 \pm a.d. 0.005,$$

and this mean I shall use in comparing this series with the later ones, omitting those results obtained at temperatures between  $0^\circ$  and  $100^\circ$ , because the want of certainty in the temperature measurements would render the labor of the necessary computations fruitless.

TABLE I.

No.	$\frac{\eta_1}{\eta_2}$	$d$ .
2	1.083	— .151
7	1.212	— .022
8	1.206	— .028
9	1.215	— .019
11	1.272	+ .038
12	1.267	+ .033
13	1.271	+ .037
14	1.273	+ .039
18	1.277	+ .043
21	1.259	+ .025
	1.234	0.044

TABLE II.

No.	$\frac{\eta_1}{\eta_2}$	$d$ .
11	1.272	+ .002
12	1.267	— .003
13	1.271	+ .001
14	1.273	+ .003
18	1.277	+ .007
21	1.259	— .011
	1.270	0.005

*Second Series.* April, 1878. — In making a study of the best forms of apparatus, a careful trial was given to capillaries in the form of a helix, as these facilitate the maintenance of constant and uniform temperatures of the bath, and render the use of longer and larger tubes possible. The coils were successfully made by winding the tubing, as it was drawn from the tube of the glass-worker, upon a wooden drum. The second series was made with a pair of such coils, and consisted of only eight measurements. The results in the measurements of  $K$  were so wholly discordant, that they were at once rejected.

*Third Series.* — A second pair of smaller coils was mounted in a manner similar to that just described, and with the same general results, showing the coils to be utterly useless; probably, as was to

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mean, and  $\frac{d_1 + d_2 + d_3 + \dots + d_n}{n}$  will be the average deviation, if, in the summation of the numerator, the values of  $d_1$ ,  $d_2$ , &c. be taken arithmetically, without regard to algebraic sign.

be anticipated, because of changing curvature of the coils owing to expansion or change of position. Several straight capillaries were also studied, varying in size and in the form of tube employed for heating the gas. These experiments covered the whole 126 measurements of this series, and resulted in the selection of the form and size given in the general description of the apparatus at page 2. No systematic study was made, however, of the effects of the proportions of the tube on the law of the transpiration, though the apparatus seems well adapted for that purpose.

*Fourth Series.* October, November, and December, 1878. — The capillaries used in this series were those described in the general account of the apparatus at page 2; viz. straight tubes of about 30 cm. in length and 0.0110 cm. in diameter, and were cut from tube *I* of the apparatus of 1876. The mercury columns were read to 0.1 mm. by a reading telescope from steel millimeter scales placed behind the gauges. Corrections were applied to these scales, but the precision of reading was much less than in the fifth series; the temperature measurements were also somewhat less precise, and the whole series should receive much less weight, owing to the conditions under which it was taken, than the fifth series on air, or the second series on carbonic acid. Table III gives the results obtained.

In the Table the letters heading the columns have the same meaning as elsewhere throughout the paper. The pressures  $p_1$ ,  $p_2$ , &c. are given in millimeters of mercury at  $0^\circ$  C. at Boston. The column headed “Mean  $\frac{\eta_2}{\eta_1}$  or  $K$ ,” contains the mean of the group of values of  $\frac{\eta_2}{\eta_1}$ , or of  $K$ , immediately preceding, and a similar statement is true of the column headed “Mean  $t_2$ .”

The results from No. 49 onwards are of much greater precision than those preceding, chiefly from the use of greater pressures, which increased the precision of measurement. Measurements 49 to 99, when combined by the method of least squares, give for constants in the empirical equation

$$\frac{\eta_t}{\eta_0} = \frac{\eta_2}{\eta_1} = 1 + At + Bt^2$$

the values, between the limits  $t = 0^\circ$  and  $t = 100^\circ$ ,

$$A = 0.002821, \text{ and } B = -0.00000149.$$

These values, however, I have regarded as entitled to so much less weight than those of the fifth series for air, that they are given here

TABLE III. — AIR. FOURTH SERIES.

No.	$t_1$ .	$t_2$ .	Mean $t_2$ .	$\eta_2$ $\eta_1$	Mean $\eta_2$ or $K$ .	$K$ .	$p_1$ .	$p_2$	$p_3$ .
	$^{\circ}$	$^{\circ}$	$^{\circ}$				mm.	mm.	mm.
1	21.4	21.4	....	....	....	1.0571	753.8	639.3	507.8
2	100.0	100.0	....	....	....	1.0471	753.8	638.9	505.3
3	99.94	0.0	....	1.265	1.0521	....	758.5	....	....
4	99.94	0.0	....	1.272	....	....	758.5	610.2	511.8
5	99.94	0.0	....	1.265	....	....	758.5	610.1	510.8
6	99.94	0.0	....	....	....	....	758.5	610.1	510.8
7	99.94	0.0	....	1.266	....	....	758.5	546.0	382.9
8	99.94	0.0	....	1.266	....	....	758.5	546.0	382.9
					1.266				
9	0.0	0.0	....	....	....	1.0542	764.3	645.0	506.5
10	0.0	0.0	....	....	....	1.0574	764.2	645.2	507.6
11	0.0	0.0	....	....	....	1.0585	764.0	645.1	507.8
12	0.0	0.0	....	....	....	1.0517	764.2	606.6	403.1
13	0.0	0.0	....	....	....	1.0520	764.3	569.8	279.3
					1.0548				
21	0.0	16.46	....	1.0308	....	....	763.4	698.3	623.5
22	0.0	16.48	....	1.0380	....	....	763.4	698.5	623.4
23	0.0	16.50	....	1.0355	....	....	763.4	698.2	622.9
			16.48		1.0348				
24	0.0	28.19	....	1.0704	....	....	762.5	699.2	620.5
25	0.0	28.32	....	1.0718	....	....	762.5	699.3	620.6
26	0.0	29.02	....	1.0655	....	....	761.9	698.2	619.1
27	0.0	29.18	....	1.0679	....	....	761.5	697.9	618.7
28	0.0	28.98	....	1.0707	....	....	761.5	698.3	619.5
			28.74		1.0693				
29	0.0	30.49	....	1.0744	....	....	761.5	698.6	619.5
30	0.0	30.41	....	1.0735	....	....	761.5	698.6	619.6
31	0.0	30.30	....	1.0774	....	....	761.4	698.2	618.5
32	0.0	30.12	....	1.0810	....	....	761.4	699.0	620.2
33	0.0	30.06	....	1.0737	....	....	761.4	698.8	620.3
			30.28		1.0760				
34	0.0	37.05	....	1.0945	....	....	761.0	699.6	619.3
35	0.0	37.15	....	1.1016	....	....	760.9	699.9	619.6
36	0.0	37.30	....	1.1035	....	....	760.8	700.2	620.3
37	0.0	37.35	....	1.0982	....	....	760.8	700.2	620.7
			37.21		1.0995				
38	0.0	53.40	....	1.1283	....	....	755.8	698.1	616.3
39	0.0	53.60	....	1.1277	....	....	755.8	698.1	616.3
40	0.0	53.96	....	1.1404	....	....	755.8	697.9	614.7
41	0.0	54.11	....	1.1456	....	....	755.8	697.8	614.0
			53.77		1.1355				
42	0.0	46.28	....	1.1318	....	....	755.3	695.4	611.8
43	0.0	46.68	....	1.1180	....	....	755.0	693.9	609.4
			46.48		1.1249				
44	0.0	56.26	....	1.1432	1.1432	....	754.8	695.4	608.9
45	0.0	68.03	....	1.1740	1.1740	....	754.8	697.4	608.5
46	0.0	66.93	....	1.1725	1.1725	....	754.8	697.2	608.1
47	0.0	67.88	....	1.1752	....	....	755.0	697.0	607.0
48	0.0	67.83	....	1.1748	....	....	755.0	696.8	606.5
			67.86		1.1750				
Tube I. remounted									
49	0.0	0.0	....	....	....	....	759.2	557.0	238.0
50	0.0	0.0	....	....	....	....	759.2	557.0	238.0
51	0.0	0.0	....	....	....	1.0517	759.3	556.7	237.5
52	0.0	0.0	....	....	....	1.0520	759.3	556.5	236.7
53	0.0	0.0	....	....	....	1.0517	759.9	556.1	232.9
54	0.0	0.0	....	....	....	1.0522	759.9	556.1	232.8
					1.0519				

TABLE III.—*Continued.*

No.	$t_1$ .	$t_2$ .	Mean $t_2$ .	$\frac{\eta_2}{\eta_1}$	Mean $\frac{\eta_2}{\eta_1}$ or $K$ .	$K$ .	$p_1$ .	$p_2$ .	$p_3$ .
							mm.	mm.	mm.
55	0.0	42.80	....	1.1202	....	....	761.4	586.7	232.9
56	0.0	42.84	....	1.1200	....	....	761.5	586.6	232.0
57	0.0	42.84	....	1.1187	....	....	761.6	586.7	232.9
58	0.0	42.95	....	1.1216	....	....	762.0	587.1	231.7
59	0.0	42.95	....	1.1216	....	....	762.0	587.1	231.7
			42.88		1.1204				
60	0.0	0.0	....	....	....	1.0526	760.4	556.6	234.1
61	0.0	0.0	....	....	....	1.0506	760.3	556.6	233.4
62	0.0	0.0	....	....	....	1.0535	760.1	555.1	228.5
63	0.0	0.0	....	....	....	1.0524	760.1	555.2	228.4
64	0.0	0.0	....	....	....	1.0498	759.9	555.6	229.1
65	0.0	0.0	....	....	....	1.0504	759.9	555.6	229.4
66	0.0	0.0	....	....	....	1.0507	759.9	555.5	229.1
					1.0514				
67	0.0	72.55	72.55	1.1913	1.1913	....	760.8	602.3	229.6
68	0.0	12.57	....	1.0372	....	....	754.3	559.5	221.5
69	0.0	12.67	....	1.0366	....	....	754.3	559.3	220.6
70	0.0	12.77	....	1.0371	....	....	754.3	559.4	220.6
71	0.0	12.92	....	1.0368	....	....	754.2	559.3	220.3
72	0.0	12.92	....	1.0365	....	....	754.1	559.2	220.3
73	0.0	12.92	....	1.0365	....	....	754.1	559.2	220.3
			12.80		1.0368				
74	0.0	21.50	....	1.0583	....	....	753.9	565.0	221.1
75	0.0	21.15	....	1.0589	....	....	753.8	565.2	222.9
76	0.0	20.85	....	1.0599	....	....	753.7	564.3	218.5
77	0.0	20.65	....	1.0602	....	....	753.7	564.4	219.3
78	0.0	20.40	....	1.0582	....	....	753.7	563.4	215.6
			20.91		1.0591				
79	0.0	20.15	....	1.0557	....	....	753.7	563.0	215.4
80	0.0	19.80	....	1.0561	....	....	753.7	562.9	215.4
81	0.0	19.65	....	1.0541	....	....	753.7	562.6	215.3
			19.87		1.0553				
82	0.0	40.01	....	1.1100	....	....	753.7	575.9	214.0
83	0.0	39.91	....	1.1116	....	....	753.8	576.1	214.0
84	0.0	39.88	....	1.1081	....	....	753.8	576.0	214.0
85	0.0	39.93	....	1.1111	....	....	754.3	576.5	214.5
			39.93		1.1102				
86	0.0	62.64	....	1.1690	....	....	754.8	590.1	214.2
87	0.0	62.59	....	1.1699	....	....	754.8	590.1	213.8
88	0.0	62.39	....	1.1706	....	....	754.8	590.1	213.8
89	0.0	62.34	....	1.1687	....	....	754.9	590.0	213.9
90	0.0	62.34	....	1.1684	....	....	754.9	590.0	214.1
			62.46		1.1693				
91	0.0	79.92	....	1.2174	....	....	755.4	600.2	214.3
92	0.0	80.02	....	1.2159	....	....	755.4	600.1	214.3
93	0.0	79.62	....	1.2170	....	....	755.4	600.1	214.3
94	0.0	79.32	....	1.2149	....	....	755.5	599.9	214.4
95	0.0	79.32	....	1.2143	....	....	755.5	599.8	214.1
96	0.0	79.27	....	1.2134	....	....	755.6	599.8	214.2
			79.58		1.2155				
97	0.0	99.66	....	1.2655	....	....	751.2	606.0	210.3
98	0.0	99.66	....	1.2657	....	....	751.2	605.2	204.0
99	0.0	99.66	....	1.2685	....	....	751.2	608.2	502.9
			99.66		1.2666				

merely as a satisfactory and important check upon the further results of that series. The equation deduced from this latter series, I regard as best representing my measurements on dry air. It will be noticed, on comparing the two series, that the greatest deviation of the two series is at  $100^{\circ}$ , where it amounts to less than 0.4 per cent.

Measurements 1 and 2 were taken during preliminary trials of the apparatus. In 3 to 8 the first capillary was in boiling water, the second in ice, this reversal of the usual sequence of temperatures being made in order to test the apparatus. The mean of these five measurements (the sixth having been rejected when made, as having been taken before the static condition of the process was reached) gives

$$y = \frac{\eta_{100}}{\eta_0} = 1.267 \pm a.d. 0.002.$$

The above equation gives for  $t = 99^{\circ}.94$

$$\frac{\eta_t}{\eta_0} = 1.2666,$$

and the direct observation made, as were all those from which the equation is derived, with the first capillary at  $0^{\circ}$ , gives as the mean at  $99^{\circ}.66$

$$\frac{\eta_t}{\eta_0} = 1.2666.$$

The close agreement of these numbers seems to indicate a freedom from constant error of any considerable magnitude in the value of  $K$  and in the expansion correction to this value.

*Fifth Series.* April, 1880.—This series was taken with the apparatus as described at page 2, immediately after the completion of the second series with carbonic acid. The measurements were made under as favorable conditions as any of this paper, and are of greater weight than all of the others upon air. The full data for the computation of these results are given in Table IV. in such form that a recomputation, in so far as relates to those portions subject to possible modification of such amount as to materially change the result, may at any time be made. The value of  $K$  used was the same as that used for the second series of carbonic acid.

These measurements may be divided according to temperatures into six groups, and are very concordant except that the sixth group shows a deviation of about 0.6 per cent from the value which the other five groups would indicate at that temperature. There appears

TABLE IV.—AIR. FIFTH SERIES.

No.	$p_1-p_2$	$p_1+p_2$	$p_2-p_3$	$p_2+p_3$	$\alpha$	$t_2$	$\frac{\eta_2}{\eta_1}$	Computed $\frac{\eta_2}{\eta_1}$	$d$
1	92.31	1418.23	113.29	1212.63	.0036693	13.70	1.0374		
2	92.26	1417.90	113.37	1212.27	.....	13.93	1.0378		
3	92.15	1417.71	113.35	1212.21	.....	14.21	1.0378		
					Means =	13.947	1.0377	1.0383	-.0006
4	83.93	1423.93	122.09	1217.91	.0036694	42.60	1.1180		
5	83.86	1423.50	122.13	1217.51	.....	43.04	1.1178		
6	83.81	1423.35	122.27	1217.27	.....	43.30	1.1187		
7	83.98	1422.78	122.41	1216.39	.....	43.17	1.1178		
8	83.98	1422.44	122.25	1216.21	.....	42.76	1.1179		
					Means =	42.974	1.1180	1.1176	+.0004
9									
10	77.39	1425.25	128.88	1218.98	.0036694	67.74	1.1861		
11	77.34	1425.16	128.76	1219.06	.....	67.74	1.1859		
12	77.34	1425.08	128.83	1218.91	.....	67.94	1.1831		
					Means =	67.81	1.1850	1.1850	.0000
13	72.44	1429.96	133.48	1224.04	.0036695	88.67	1.2381		
14	72.29	1430.11	133.59	1224.23	.....	88.64	1.2419		
15	72.20	1430.14	133.49	1224.45	.....	88.85	1.2419		
16	72.20	1429.96	133.59	1224.17	.....	88.92	1.2424		
					Means =	88.77	1.2411	1.2415	-.0004
17	70.20	1441.48	136.41	1234.87	.0036696	99.69	1.2684		
18	70.28	1441.22	136.35	1234.59	.....	99.02	1.2686		
19	70.23	1440.51	136.58	1233.70	.....	98.93	1.2716		
20	70.05	1440.43	136.11	1234.27	.....	99.41	1.2695		
21	70.05	1440.19	136.14	1234.00	.....	99.03	1.2710		
22	70.10	1440.10	136.16	1233.84	.....	99.18	1.2695		
					Means =	99.21	1.2698	1.2696	+.0002
23	64.94	1446.20	141.17	1240.09	.0036697	124.61	1.3321		
24	64.86	1446.46	141.21	1240.39	.....	124.36	1.3350		
25	65.12	1446.42	140.95	1240.35	.....	124.17	1.3279		
26	64.98	1446.72	141.02	1240.70	.....	124.22	1.3314		
27	65.12	1446.94	140.96	1240.86	.....	124.69	1.3264		
					Means =	124.41	1.3306	1.3368	-.0062

to be in this group some considerable constant error, due probably to a mistake in the reading of the thermometer which was a different one, from that used in the preceding groups. The deviation is so large that I have thought it best to reject the sixth group wholly in the computation of the empirical equation for this series on air. I fully recognize that this rejection may be considered somewhat arbitrary, and also that the retention of the sixth group would make the accordance between the fourth and fifth series greater than now; but I am so fully satisfied that there is some mistake in the rejected group

that I unhesitatingly lay it aside. I thus obtain by the method of least squares for measurements 1 to 22 on dry air, 5th series,

$$\frac{\eta_t}{\eta_0} = 1 + 0.002751 t - 0.00000034 t^2,$$

between the temperature limits of  $0^\circ$  and  $100^\circ\text{C}$ . The last column of the table, headed  $d$ , gives the deviations of the observed means from the values of  $\frac{\eta_t}{\eta_0}$ , computed for the same temperatures  $t_2$  by the preceding equation and given in the ninth column. The average deviation of the means from the equation is thus only 0.0003, or about 0.03 per cent, for the first five means, the deviation of the sixth mean being ten times the next greatest deviation. The average deviation of the individual observations from their respective means, disregarding the correction corresponding to the slight differences of the observed temperatures from the mean, is 0.0012 or about 0.1 per cent.

#### *Carbonic Acid.* $\text{CO}_2$ .

*First Series.* — 21 measurements were made in January, 1879, after the close of the fourth series on air, and with the same apparatus, with the addition of the necessary apparatus for holding the gas. The precision of the results was less than in the fourth series for air, chiefly because of the somewhat greater complication of the apparatus for the supply of the gas. The results are of little value, but are given as a check on the second series. The pressures were about the same as in Nos. 49 to 98 of the fourth series on air. The temperature  $t_1$  of the first capillary was always  $0^\circ\text{C}$ . The following Table V. gives the mean results. The coefficient of expansion used was 0.003699, a value taken from Regnault's work by interpolation for the actual pressure  $p_2$ , under which the expansion occurs.

The column headed  $n$  gives the number of measurements going to make up the corresponding means.

TABLE V. —  $\text{CO}_2$ . FIRST SERIES.

$n$ .	$t_2$ .	$\frac{\eta_2}{\eta_1}$	Computed $\frac{\eta_2}{\eta_1}$
8	29.61	1.103	1.108
5	48.56	1.178	1.175
8	100.09	1.324	1.351

*Second Series.* — 120 measurements were made from January to April, 1880, with the complete apparatus as described at pages 2 and 9. The thermometers used were Casella 32378, Baudin 7335 up to  $180^{\circ}$ , and Baudin 7789 above  $200^{\circ}$ . The first 20 measurements were preliminary, being made with various tubes and forms of apparatus before satisfactory forms were obtained. The remainder are given in the following table. In all cases  $t_1 = 0^{\circ}$ .

TABLE VI.—CO<sub>2</sub>. SECOND SERIES.

No.	$p_1$ .	$p_2$ .	$p_3$ .	$K$ .	Mean $K$ .
1	784.08	720.53	653.77	1.0422	1.0440
2	786.69	720.17	653.99	1.0918	
3	787.48	721.58	654.10 •	1.0737	
4	785.44	722.30	654.54	1.0204	
5	790.61	724.59	654.84	1.0397	
6	758.59	700.78	640.07	1.0364	
7	758.17	700.30	639.71	1.0638	
8	757.60	699.82	639.23	1.0380	
9	757.15	699.27	638.57	1.0381	
10	773.74	707.56	637.41	1.0390	
11	774.14	707.50	636.92	1.0169	
12	780.55	718.34	652.49	1.0330	
13	780.74	718.12	652.18	1.0387	
14	748.64	699.58	649.47	1.0510	
15	748.45	699.99	649.37	1.0276	
16	748.27	699.72	649.27	1.0330	
17	748.52	699.55	648.81	1.0365	
18	748.37	699.34	648.75	1.0408	
19	748.19	699.33	648.68	1.0359	
20	771.58	614.35	409.82	1.0402	1.0375
21	771.62	614.36	409.70	1.0376	
22	771.66	614.35	409.57	1.0399	
23	771.67	614.28	409.22	1.0394	
24	772.11	614.50	409.16	1.0397	
25	772.38	614.57	408.98	1.0401	
26	770.73	611.24	400.91	1.0353	1.0395
27	770.55	610.67	400.83	1.0404	
28	770.37	610.55	400.81	1.0404	
29	771.92	701.15	625.09	1.0335	
30	771.71	700.74	624.68	1.0366	
31	771.50	700.54	624.90	1.0419	
32	767.36	705.39	640.08	1.0386	
33	767.46	705.46	640.22	1.0394	
34	766.90	705.13	639.89	1.0362	
35	766.80	705.01	639.71	1.0357	
36	767.34	712.33	655.13	1.0406	
37	767.29	712.39	655.18	1.0383	
38	766.95	712.12	654.91	1.0370	1.0380



TABLE VI. — CO<sub>2</sub>. SECOND SERIES. — *Continued.*

No.	$p_1$ .	$p_2$ .	$p_3$ .	100 $\alpha$ .	$t_2$ .	$\frac{\eta_2}{\eta_1}$ .	Mean $t_2$ .	Mean $\frac{\eta_2}{\eta_1}$ .	Computed $\frac{\eta_2}{\eta_1}$ .
39	746.72	693.79	630.67	0.37047	17.62	1.0688	18.10	1.0679	1.0666
40	746.53	693.62	630.49		17.87	1.0684			
41	746.18	693.31	630.10		18.09	1.0696			
42	745.93	692.94	629.77		18.29	1.0658			
43	745.64	692.71	629.47		18.62	1.0669			
44	745.41	696.50	628.79	0.37053	41.15	1.1468	40.98	1.1458	1.1485
45	745.64	696.68	629.03		41.28	1.1472			
46	745.97	697.01	629.24		40.86	1.1481			
47	746.41	697.32	629.70		40.86	1.1426			
48	746.73	697.75	630.22		40.87	1.1438			
49	747.45	698.54	630.98		40.87	1.1461			
50	769.40	723.90	653.79	0.37071	59.02	1.2125	59.14	1.2126	1.2119
51	769.56	724.17	653.75		59.30	1.2197			
52	769.49	724.18	654.04		59.54	1.2164			
53	769.47	724.20	654.31		59.50	1.2163			
54	769.25	723.90	654.43		58.93	1.2064			
55	769.23	723.89	654.25		58.72	1.2102			
56	769.26	724.01	654.68		58.96	1.2068			
57	774.14	732.22	659.65	0.37077	79.56	1.2847	79.65	1.2848	1.2820
58	774.07	732.31	659.96		79.58	1.2860			
59	773.95	732.35	660.33		79.72	1.2857			
60	773.66	732.17	660.48		79.73	1.2829			
61	773.71	735.84	662.96	0.37081	99.67	1.3550	100.17	1.3506	1.3508
62	773.62	735.83	663.22		100.17	1.3514			
63	773.50	735.77	663.52		100.24	1.3468			
64	773.78	736.13	663.86		100.60	1.3490			
65	780.77	745.42	670.80	0.37087	119.11	1.4147	119.30	1.4148	1.4139
66	780.70	745.31	670.68		119.20	1.4129			
67	780.32	745.04	670.69		119.27	1.4120			
68	779.81	744.65	670.56		119.37	1.4119			
69	779.58	744.58	670.24		119.55	1.4223			
70	770.66	738.34	663.10	0.37083	141.53	1.4774	141.893	1.4843	1.4873
71	770.52	738.18	662.91		141.89	1.4757			
72	770.55	738.38	663.11		141.90	1.4836			
73	770.43	738.52	663.31		141.87	1.4948			
74	770.18	738.28	663.31		141.92	1.4919			
75	769.77	737.82	663.10		142.25	1.4824			
76	766.48	734.44	652.66	0.37078	158.40	1.5492	158.283	1.5371	1.5400
77	766.22	734.26	652.58		158.83	1.5498			
78	766.06	734.26	653.19		159.44	1.5446			
79	765.38	733.54	652.85		159.05	1.5415			
80	763.89	732.12	652.69		157.87	1.5218			
81	763.60	731.91	652.61		157.56	1.5244			
82	763.28	731.73	652.46		157.62	1.5341			
83	763.05	731.56	652.45		157.50	1.5310			

TABLE VI.—CO<sub>2</sub>. SECOND SERIES.—*Continued.*

No.	$p_1$	$p_2$	$p_3$	100 $a$ .	$t_2$	$\frac{\eta_2}{\eta_1}$	Mean $t_2$ .	Mean $\frac{\eta_2}{\eta_1}$ .	Computed $\frac{\eta_2}{\eta_1}$ .
84	773.83	745.39	666.15	0.37088	181.49	1.6141	181.32	1.6188	1.6185
85	773.96	745.70	666.23		180.99	1.6309			
86	773.97	745.67	666.23		180.96	1.6280			
87	774.14	745.64	666.21		181.70	1.6136			
88	774.01	745.49	666.38		181.45	1.6072			
89	774.41	747.71	660.30	0.37090	223.9	1.7284	224.0	1.7474	1.7488
90	774.32	747.94	661.30		225.0	1.7312			
91	773.99	747.87	660.78		223.4	1.7631			
92	774.33	748.38	661.66		223.7	1.7669			
93	771.47	722.92	666.03	0.37071	16.20	1.0668	17.07	1.0630	1.0628
94	770.89	722.21	665.19		16.62	1.0646			
95	769.33	677.80	561.76	0.37036	17.18	1.0602			
96	769.14	677.75	561.63		17.30	1.0621			
97	768.79	677.40	561.21		17.47	1.0620			
98	768.73	677.38	561.12		17.62	1.0625			

Measurements 39 to 98 furnish data for computing the constants in an empirical equation for carbonic acid, and the equation thus deduced is

$$\frac{\eta_t}{\eta_0} = 1 + 0.003725 t - 0.00000264 t^2 + 0.0000000417 t^3,$$

between the limits of 0° and 224° C.

The term containing  $t_3$  is necessary on account of the rapid curvature of the line representing the observations. The last column of Table VI. shows the values of  $\frac{\eta_2}{\eta_1} = \frac{\eta_t}{\eta_0}$  deduced from this equation for the temperature means of the second column, showing as close an agreement with the observed means of  $\frac{\eta_2}{\eta_1}$  as could be expected except at 100°, where the deviation amounts to about two per cent. The first series is however of much less weight than the second series. The last column of Table VI. shows the values of  $\frac{\eta_2}{\eta_1}$  computed to correspond with the means of the preceding column. The average deviation of the observed means from those computed from the above equation is thus 0.0020 or about 0.15 per cent. The average deviation of the individual results from their corresponding means, taken as in the fifth series for air, is 0.0052, or about 0.4 per cent.

*Effect of Pressure.*

In Series IV. for air, the total driving pressure used,  $p_1 - p_3$ , was varied within considerable ranges, to test whether the value obtained of  $\eta_2 : \eta_1$ , or of  $K$ , was thereby affected except through errors of observation. In 3, 4, and 5,  $p_1 - p_3 = 248$  mm., and  $\eta_2 : \eta_1 = 1.267$ ; in 7 and 8, under the same conditions otherwise,  $p_1 - p_3 = 376$  mm., and  $\eta_2 : \eta_1 = 1.266$ , showing no effect due to increase of the pressure by one half. Measurements 9 to 13, made with  $p_1 - p_3$  ranging from 258 to 485 mm., give values of  $K$  with an average deviation of about 0.2 per cent. Measurements 21 to 48 give values of  $\eta_2 : \eta_1$  about one per cent lower than those obtained in the remainder of the series, made with greater values of  $p_1 - p_3$ , but the precision of these measurements is much less than of the later ones. The value of  $K$  found from 50 to 54 and 60 to 66, with  $p_1 - p_3 = 530$  mm. about, is within 0.5 per cent of that found in 9 to 12, under  $p_1 - p_3 = 258$  mm., and agrees precisely with 12, under  $p_1 - p_3 = 361$  mm. And, finally, the mean of 97 and 98 differs from 99 by less than 0.25 per cent, though the pressures  $p_1 - p_3$  are respectively 540 and 250 mm.

In the second series for carbonic acid, the effect of pressure was somewhat tested in measurements 26 to 38, where  $p_1 - p_3$  varied from 370 to 111 mm. without producing any traceable effect in  $K$ . This is the best test made, owing to the greater precision of the measurements. Also in measurements 93 to 98 of the same series the pressure was changed from 105 to 207 without materially affecting the results.

In the first series for air, the average value of  $p_1 - p_3$  was about 740 mm., and the mean value of  $\eta_2 : \eta_1$  at  $t_2 = 100^\circ$  is 1.270, while that of the fourth series, under  $p_1 - p_3 =$  from 248 to 540, is about 1.267, and that of the fifth series, under  $p_1 - p_3 = 225$  mm., is 1.272, thus showing no effect traceable to the difference of the driving pressures used.

*Deduction from Results.*

The results of my measurements seem to show conclusively, that the variation of the viscosity with the temperature of the gas, in the case at least of dry carbonic acid and of dry air freed from carbonic acid, which may be taken as typical gases, is not proportionate either to the square root or to any numerical power of the absolute temperature reckoned from  $-273^\circ \text{C}$ . They point thus to the inference that all hypotheses yet advanced to account for the variation of the viscosity

of gases, and hence also for the viscosity itself, are incomplete for this phenomenon, in the same general way as are the analogous hypotheses regarding the compressibility of gases and other phenomena. The hypothetical deductions fail to accord completely with the results of quantitative measurements.

#### DISCUSSION OF RESULTS OF ALL RESEARCHES ON AIR AND CO<sub>2</sub>.

The experimental results which I have given in the foregoing tables, and the deduced equations, show that, unless some considerable and regular source of error affects them, the viscosity of both dry air (freed from CO<sub>2</sub>) and carbonic acid increases with the rise of temperature according to a rate which varies with the temperature and is smaller as the temperature is higher. The ratio  $\eta_t : \eta_0$  is therefore not proportional to the first power (Maxwell) of the absolute temperature; it is not expressible by a linear equation of the first degree, e. g.  $\eta_t = \eta_0 (1 + b t)$ , where  $t$  = temperature C. and  $b$  = a constant; nor is it given by the equation  $\eta_t = \eta_0 (1 + a t)^n$ , or its equivalent  $\eta_t \div \eta_0 = c \tau^n$ , where  $\tau$  = absolute temperature. The last three equations are the only ones thus far used by observers in discussing their results, although the work of E. Wiedemann and A. von Obermayer shows a decided departure from these expressions. The insufficiency of the latter equations may be shown by discussing the observational data by the logarithmic method which I used in my former paper. Nor could the equation  $\eta_t = \eta_0 (1 + a t)^n$  be more than an empirical equation at best, unless possibly when  $a$  were expressed as a function of  $t$  and the pressure, a condition which I have not thought worth consideration at present. For the expression of my own results, I have employed merely the empirical equation with increasing powers of  $t$ , as I have found no theoretical hypothesis which led to results corresponding to the observed relation of  $\eta$  and  $t$ .

The deviation of both air and carbonic acid from the equation  $\eta_t = \eta_0 (1 + b t)$  is so small, and the difficulties of precise measurement so considerable, that these deviations may often be masked by accidental errors of measurement, and by "constant errors," arising from imperfect drying or purification of the gas, from differences between the thermometer indications and the actual temperature of the gas, from faulty proportioning of apparatus, and from other sources. This discussion of my own method in these regards I give at page 25; that of others, I cannot advantageously attempt. But I

will proceed to a review of the results obtained by all others who have worked in this field; and shall show that for air and carbonic acid no results are found giving a value of  $b$  (rate of change of  $\eta_t : \eta_0$  with rise of temperature) increasing with the temperature; that some results can give only a constant value to  $b$ , owing either to the want of sufficient precision, or to the insufficient number of temperature intervals employed; that some results show a marked diminution, as do my own, of  $b$ ; and that many of these last, when carefully discussed, afford material for getting at a numerical measure of the change in  $b$  which is in substantial accordance with my own results. These facts indicate, either that all measurements show conclusively that  $b$  does diminish with rise of temperature, or that the method of transpiration through capillary tubes upon which these demonstrations rest (for the measurements with oscillating plates are of insufficient precision for determination of this change) is faulty in either its experimental application or its mathematical theory. In the consideration of this last proposition the statements which I have made at page 25 should be reviewed.

In the discussion of the results of former observers, I have adopted the graphical method as best adapted to the purpose; but as the point to be considered is one in which changes in the fourth and sometimes the fifth place of significant figures must be exhibited, a special device must be resorted to. I have therefore assumed for carbonic acid an equation  $\eta_t = \eta_0 \left(1 + \frac{t}{300}\right)$ , and for air  $\eta_t = \eta_0 \left(1 + \frac{t}{400}\right)$ , as convenient equations to which to refer the results on these gases respectively. I have computed for two or many temperatures (usually those of observation) values of  $\eta_t : \eta_0$  from these equations, and subtracting these from the experimental data for the same temperature (or from ratios  $\eta_t : \eta_0$  deduced by myself from the data), I have obtained differences or residuals which I have used as ordinates in the lines shown in the plots on Plates I. and II. These lines or "residual curves"\* easily show the fourth place of decimals in the ratio  $\eta_t : \eta_0$ , and develop as a curvature the change of the rate  $b$ . As the residual curves are plotted, convexity upwards shows a diminution of  $b$  with rise of  $t$ , concavity upwards would show an increase of  $b$ , and no curvature of course indicates a constant value of  $b$ . Above and below the  $50^\circ$ ,  $100^\circ$ , and  $200^\circ$  points of my own results will be seen a vertical row of points marked 1%, 2%, —1%, etc., which indicate dif-

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\* See Pickering, *Physical Manipulation*, i. 12; *Jour. Franklin Inst.*, lxi. 272.

ferences in  $\eta_t : \eta_0$  of 1, 2, etc. per cent from my results, and afford a convenient means of comparing the relative concordance between the results of various observers. It should be borne in mind, however, that some of the lines are plotted from means, some from single observations, and others from equations deduced either by the observers or by myself. The individual points, therefore, must not be used without qualification as a measure of the precision of the results obtained by any single observer.

*Carbonic Acid, CO<sub>2</sub>.*

The curve *OA* on Plate I. is from the equation deduced at page 17 for my own results. There can be no question as to the decided curvature shown being far beyond the limits of variable experimental error. My computations have also shown that the third power of the temperature in the linear equation was necessary. The experimental means are marked along the curve.

The curve *DOB* is from an equation of the same form

$$\eta_t = \eta_0 (1 + At + Bt^2),$$

in which *A* and *B* are computed from the results of A. von Obermayer\* with capillary "I." The term  $Ct^3$  was omitted because of the smaller number of points and their somewhat less precision. That the line represents approximately the data will be seen from noting the distribution about it of the observed points. The curvature is here also undoubted, although Obermayer merely remarks that "it follows from these experiments with some probability that the increase in coefficient of friction with rising temperature is less at higher than at lower temperatures." A comparison of this curve with mine shows that Obermayer's results and my own are in substantial agreement as to the rate of change in *b*, the curvature of the two lines being very nearly the same. Indeed, if my results be thrown into the same equation, using only the first and second powers of *t*, the value of *B* becomes 0.00000136, while that from Obermayer's data is  $B = 0.00000137$ . The value of *A* from my results is, however, considerably larger than from Obermayer's, so that his value of  $\eta_t : \eta_0$  at 100° is about 0.7 per cent below mine, and at 200° about 1 per cent below. The line *OC* is from the equation

$$\eta_t = \eta_0 (1 + 0.003582t - 0.00000105t^2),$$

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\* A. VON OBERMAYER, Wien. Ber., lxxiii. 468 (1876).

deduced by me from Obermayer's experiments at the three indicated temperatures with the capillary "D." This is in still closer agreement with mine in numerical values of  $\eta_t : \eta_0$ , but has slightly less curvature ( $B = 0.00000105$ ). The value for  $A$  in my results is 0.003637, with the same terms in the equation. When methods as different in detail as those of Obermayer and myself are in so close agreement as these appear to be when thus discussed, the chances of large constant errors other than those inherent in the use of transpiration methods seem to be much reduced.

On the line  $JH$  are three points showing the results obtained by Eilhard Wiedemann. The line is drawn from the equation

$$\eta_t = \eta_0 (1 + 0.003727 t - 0.00000320 t^2),$$

which I have deduced from these three points. The curvature is much greater than for either of the other lines, the value of  $B$  being more than twice as great as in the lines  $OA$  and  $OB$ , and three times as great as in  $OC$ . A reference to my discussion of the results for air will show that Wiedemann's results there exhibit similar relative characteristics. Up to  $100^\circ$  for  $\text{CO}_2$  the results differ numerically by less than 0.7 per cent from those of Obermayer and myself, but at  $200^\circ$  the difference is nearly 3 per cent.

The results by Puluj, as shown by the line  $OF$ , obtained by oscillating plates, are smaller than all others, a difference which characterizes his results for air. The range of temperature used is too limited to render the results available in the present consideration.

*Conclusion.*—In the absence of any really satisfactory means of assigning proper weights to the results of Obermayer, Wiedemann, and myself, it seems best to allow the results to stand in the form above given, without an attempt to deduce a mean.

### *Air.*

In Plate II. are shown the residual curves for all available observations on air. The line  $IJ$ , representing Maxwell's results, doubtless owes its steepness to some large constant error, for which several explanations have been offered. The lines for O. E. Meyer's results from capillary transpiration are

$$\begin{array}{ll} AB \text{ from Eqn. } \eta = 0.000171 (1 + 0.0024 t), & \text{Range } 20^\circ \text{ to } 99^\circ; \\ CD \text{ " " } \eta = 0.000170 (1 + 0.0028 t), & \text{" " } 21^\circ \text{ to } 100^\circ; \\ EF \text{ " " } \eta = 0.000174 (1 + 0.0030 t), & \text{" " } 21^\circ \text{ to } 100^\circ; \end{array}$$

and from oscillating plates the line is  $GH$  from the mean of the equations,

$$\begin{aligned} \eta &= 0.000186 (1 + 0.0030 t), \\ \text{and} \quad \eta &= 0.000189 (1 + 0.0025 t), \quad \text{Range } 19^\circ \text{ to } 82^\circ, \end{aligned}$$

given for different methods of computing one series of data. It is perhaps worthy of remark, that the mean of Meyer's results at  $100^\circ$  agrees with the results of my fifth series within 0.2 per cent. Meyer's observations were made chiefly at two temperatures, and thus, as well as on account of the magnitude of the errors of observation, are incapable of indicating the small deviation of  $\eta_t : \eta_0$  from the equation of the first power of  $t$ .

The observations of Puluj are shown by the lines  $KL$ ,  $MN$ ,  $OP$ ,  $QR$ , and  $ST$ , of which all but  $ST$  are from transpiration experiments, this being by oscillating plates. From the data which I have used in plotting the lines Puluj deduces the first power equations:—

$$\begin{array}{ll} KL, & \eta = 0.000179 (1 + 0.0024 t), \quad \text{Range } 13^\circ.0 \text{ to } 27^\circ.0. \\ MN, & \eta = 0.000179 (1 + 0.0023 t), \quad \text{“ } 13^\circ.6 \text{ to } 76^\circ.7. \\ OP, & \eta = 0.000181 (1 + 0.0022 t), \quad \text{“ } 1^\circ.1 \text{ to } 77^\circ.4. \\ QR, & \eta = 0.000180 (1 + 0.0021 t), \quad \text{“ } 1^\circ.5 \text{ to } 92^\circ.7. \\ ST, & \eta = \text{constant } (0.03855 + 0.00010213 t) \quad \text{“ } -3^\circ.0 \text{ to } 25^\circ.0. \end{array}$$

From the plots on Plate II., and from others which I have made, it appears to me that the data shown in  $KL$  cannot be represented by a straight line, nor are they to be relied upon for giving the rate of change of  $\eta_t : \eta_0$  at all, the curvature of the line  $KL$  which approximately represents the data being undoubtedly the result of uncorrected constant error. The points of the line  $MN$  show an upward convexity of slight amount; those of the line  $OP$  are so irregularly distributed as to conceal any slight systematic deviation from the straight line. The points of  $QR$  are also very irregularly distributed, but are unquestionably more nearly represented by the line  $QR$ , which I have drawn approximately through them, than by the straight line  $SR_1$ , which corresponds to the equation given by Puluj, or by any other straight line. The line  $QR$  should certainly be convex upwards.

The numerous experiments of Obermayer give almost unquestionable evidence that even for air the coefficient of viscosity increases at a less rapid rate at higher than at lower temperatures. The line  $aSbc$  represents the mean of results by capillaries “11,” “26,” and



"D." (I was obliged to omit "I" from consideration, owing to an apparent error. Wien. Ber., lxxiii. 440.) The separate results are fairly accordant. This line agrees quite closely with my fifth series, *SY*. The results obtained with a brass capillary are indicated by the line *de*, which is a continuation also of *Sb*, since the brass capillary agreed substantially with the others at *b*. The curvature of this line, as well as of *aSbc*, is decidedly convex upward, much more so than *SY*. Obermayer's further series with capillary "D" is of much value. It is represented in the line *gh*, and the three marked points indicate the points yielded by the experiments. This line is also decidedly convex upward, and its curvature seems to be beyond the range of variable errors of measurement. I have connected the individual points with each other by a broken line, because, owing to the small number of points, almost any form of curve could be drawn through them, and to select any one equation would be wholly arbitrary. This remark applies also to the line *UV*, representing Wiedemann's results. This line is still more convex upward than either of the preceding. The line *WX*, representing Warburg's results with oscillating plates, is in substantial accord with the results by Obermayer, Wiedemann, and myself, as well as with the mean of Meyer's results; but as it is derived from observations including but one temperature interval, it cannot serve to determine any beyond the first power variation in the coefficient.

To show the relation of the several lines to one from an equation of the exponential form which has been so generally adopted, I have drawn the line *Z* from the equation

$$\frac{\eta_t}{\eta_0} = (1 + 0.003670)^{0.76}.$$

The fifth series of my own results is shown in the line *SY*, the experimental means being denoted by crosses. The deviation of *SY* from a straight line is quite slight, and I am confident that this is a close approximation to the true result, and that the greater curvature of the lines from my own earlier results and those of other observers is due in part to impurities in the air, either in the form of carbonic acid or of vapor of water. I regard those of my measurements on air which precede the fifth series merely as checks upon the accuracy of the process, and as possessing small weight as compared with that series, for reasons assigned in the following Critique of the Method. The same is true of all but the last series with carbonic acid.

## CRITIQUE OF THE METHOD.

Since the variation to be studied in the present investigation is one of quite small amount, and is determined as the result of a somewhat complex measurement, any systematic error of even quite small magnitude might mask the quantity sought. It is essential, therefore, to show how far such errors of sensible magnitude have been avoided or eliminated. In the following pages I have given a somewhat detailed discussion of the possible sources and magnitudes of error in my measurements, and of the accuracy attained in the various component measurements.

The advantages of the method arise from its simplicity and directness. The only measurements of precision required are of the temperature of the second capillary, and of the lengths of several mercury columns. No measurements of volume are involved. The two capillaries through which identically the same masses of gas are successively transpired are as nearly as possible alike, and under the same conditions except as to temperature. The gas is transpired through each under nearly the same pressure difference, and at a pressure not far from one atmosphere. The flow of the gas is perfectly uniform, except for slight accidental fluctuations, throughout the whole of a measurement or set of measurements, tending thus to the elimination of a class of errors possible in such methods as that of oscillating plates, or of transpiration when the driving pressure falls from a given initial to a less final amount. The disturbing effect of eddies, or other special action, at the entrance to or exit from the tube, would be in part eliminated in this method, since the measurements give the ratios of the resistance in two tubes under constant and nearly identical conditions. No complete discussion of these effects has been yet given, and their experimental elimination certainly seems more easy than their mathematical treatment. It seems quite possible that they are still sensible sources of deviation from the assumed law of transpiration, but as the magnitude of the disturbance is small, and cannot be widely different in the two tubes used in this apparatus, and since the determination of the constants of the tubes dependent on their diameters, lengths, &c. are experimentally made by the same process and under conditions identical with the subsequent work, the resulting error must be small. It should be noted, also, that the debated question as to the slip of the gas over the inner surface of the glass tube has an influence in my results only to the extent by which this slip is affected by the temperature.

The freedom of the method from constant error is indicated by the close accordance of the results on dry air as obtained by the first, fourth,

and fifth series. The first series was made with an apparatus totally different from that afterwards used in all details of disposition of parts and of instruments used, except that the tube *I* of that series was the same that was afterward cut into three pieces, of which two nearly equal ones served as the capillaries in the fourth and fifth series. Observations with other tubes in the preliminary measurements, however, checked satisfactorily with these. The instruments and arrangements for measuring the pressures in the fourth series were wholly different from those used in the fifth series. The results at 100° for the three series are given in the following Table VII., in which the last two values are taken from the equation deduced for the respective series.

TABLE VII.

Series.	$\frac{\eta_{100}}{\eta_0}$	<i>d.</i>
I.	1.270	—0.0017
IV.	1.2672	—0.0045
V.	1.2717	0.

The quantity deduced as the result of each experiment of the present investigation is a ratio between the coefficients of viscosity of a gas at two different measured temperatures. As I have already shown,\* this ratio is expressible by the following equation :

$$y = \frac{\eta_t}{\eta_0} = \frac{R_2^4 \lambda_1}{R_1^4 \lambda_2} \cdot \frac{p_2^2 - p_3^2}{p_1^2 - p_2^2} \cdot \frac{1 + 3 A t}{1 + \alpha t},$$

where

$\eta_t$  = coefficient of viscosity of the gas at  $t^\circ$  C.

$\eta_0$  = " " " "  $0^\circ$  C.

$R_2$  = radius, and  $\lambda_2$  = length, of second capillary at  $t^\circ$ .

$R_1$  = radius, and  $\lambda_1$  = length, of first " "  $0^\circ$ .

$p_1$  = pressure of gas at entrance to first capillary.

$p_2$  = " " exit from first capillary, which is the same as that at entrance to second.

$p_3$  = pressure at exit from second capillary.

$t$  = temperature of second capillary.

$0^\circ$  = " first "

$A$  = coefficient of linear expansion of the glass.

$\alpha$  = mean coefficient of expansion of the gas between  $0^\circ$  and  $t^\circ$ , and under the pressure  $p_2$ .

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\* See references at end of paper.

Writing for brevity,

$$\frac{R_2^4 \lambda_1}{R_1^4 \lambda_2} = K, \text{ a constant;}$$

$$\frac{p_2^2 - p_3^2}{p_1^2 - p_2^2} = \phi(p), \text{ a function of } p;$$

and

$$\frac{1 + 3At}{1 + at} = \psi(t), \text{ a function of } t;$$

there results,

$$y = K \cdot \phi(p) \cdot \psi(t).$$

Differentiating this expression successively with respect to  $K, p_1, p_2, p_3, t, A$ , and  $a$ , then substituting for  $dy, dK, dp_1$ , &c.  $\Delta y, \Delta K, \Delta p_1$ , &c., and solving with respect to  $\Delta K, \Delta p_1$ , &c., the following expressions may be obtained:

$$\begin{aligned}\Delta K &= \frac{1}{\phi(p) \cdot \psi(t)} \cdot \Delta y \\ \Delta p_1 &= -\frac{(p_1^2 - p_2^2)^2}{2p_1(p_1^2 - p_2^2)} \cdot \frac{1}{K \cdot \psi(t)} \cdot \Delta y \\ \Delta p_2 &= \frac{(p_1^2 - p_2^2)^2}{2p_2(p_1^2 - p_2^2)} \cdot \frac{1}{K \cdot \psi(t)} \cdot \Delta y \\ \Delta p_3 &= -\frac{p_1^2 - p_2^2}{2p_3} \cdot \frac{1}{K \cdot \psi(t)} \cdot \Delta y \\ \Delta t &= -\frac{(1 + at)^2}{a} \cdot \frac{1}{K \cdot \phi(p)} \cdot \Delta y \\ \Delta A &= \frac{1 + at}{3t} \cdot \frac{1}{K \cdot \phi(p)} \cdot \Delta y \\ \Delta a &= -\frac{(1 + at)^2}{(1 + 3At)t} \cdot \frac{1}{K \cdot \phi(p)} \cdot \Delta y.\end{aligned}$$

From these, by inserting the proper numerical quantities taken from any suitable experimental result, may be computed the numerical values of  $\Delta K, \Delta p_1$ , &c., corresponding to any assumed value of  $\Delta y$ , and thus may of course be found the amount of change (or error) in  $K, p_1$ , &c., which would produce in  $y$  the assumed change (or error)  $\Delta y$ .

For the present purposes it is convenient to assume for  $\Delta y$  the same fractional value at all temperatures, and a convenient and suitable magnitude is  $\Delta y = 0.001 y$ . The following Table VIII. gives the

numerical values of  $\Delta K$ ,  $\Delta p$ ,  $\Delta t$ , &c., deduced from the experimental data of the second series for carbonic acid. The first column gives the tabular number of the measurements from which the data were taken.

TABLE VIII.  $\Delta y = 0.001 y$ .

No.	$t_2$ .	$\Delta K$ .	$\Delta p_1$ .	$\Delta p_2$ .	$\Delta p_3$ .	$\Delta t$ .	$\Delta A$ .	$\Delta a$ .	$\Delta H$ .
	°		mm.	mm.	mm.	° per ct.	per ct.	per ct.	
50	59.0	0.00104	-0.043	+0.028	-0.074	-0.33 - .56	+0.0000057 +70	-0.000021 .57	7.6
62	100.2	0.0010	-0.037	0.025	-0.077	-0.37 - .37	0.0000033 40	-0.000014 .40	
82	157.6	0.0010	-0.031	0.023	-0.084	-0.43 - .43	0.0000021 26	-0.000010 .27	
90	225.0	0.0010	-0.026	0.020	-0.092	-0.50 - .50	0.0000015 19	-0.0000082 .22	2.5

*Probable Effect on  $y$  of Instrumental Errors and Errors of Observation on the Values found and used for  $K$ ,  $p_1$ ,  $p_2$ , etc.*

Since the accuracy in the determination of  $K$  depends upon that of  $p_1$ ,  $p_2$ , and  $p_3$ , when both tubes are in ice, and thus under identical conditions, the latter quantities will be first considered.

I.  $\Delta p$ .

*Constant errors in  $p_1$ ,  $p_2$ ,  $p_3$* , may be due to instrumental error of the barometer, or of the cathetometer which is used in the measurement of the mercury columns. Of course the fractional precision of the pressure measurements would be increased by increasing the driving pressure, but the possible deviation from the assumed laws of transmission of the gas through the tubes and of the increased liability to leakage with great rarefaction renders the use of a small pressure desirable. The cathetometer error has been reduced by the application of corrections determined for each decimeter of the scale by Prof. Wm. A. Rogers of Cambridge, and it was found not difficult to read the length of the mercury columns in the gauges with a probable error of less than 0.02 mm. The barometer was No. 835, made by James Green of New York, and had a scale of millimeters with vernier reading to 0.1 mm., and read by estimation as nearly to 0.01 as possible. Its instrumental error after careful refilling was determined by tests with the cathetometer and by indirect comparison with instruments of the U. S. Signal Service, and was probably under 0.3 mm., possibly considerably under that amount. As will be seen farther on, a constant error of this magnitude would have no sensible effect.

Further constant errors may be due to the following causes:—

1. Impure mercury in gauges. Carefully redistilled mercury only was employed. Any slight error from this source would be nearly eliminated through the use of the same mercury in the determination of both  $\gamma$  and  $K$ .

2. Imperfect drying of the gas. The gas was retained over concentrated sulphuric acid from fifteen minutes upward before being passed through the drying tubes into the apparatus. These tubes were of more than 0.5 meter in length, and of such size that their contents would supply the apparatus with gas for upwards of a quarter of an hour. They contained anhydrous phosphoric acid, of which the surface was made greater by placing a portion of it upon a horizontal partition of wire gauze. The apparatus was dried as thoroughly as possible before the beginning of each day's observations, by repeated careful exhaustions, and by the transpiration of the dried gas during several hours.

3. Leakage into the apparatus in the portion where  $p_1$  was measured would not occur, as this pressure was always equal to or in excess of the external. Leakage into the space containing the gas under  $p_2$  would be fatal to accuracy, and was prevented at the necessary joints by placing as close together as practicable the ends of the glass tubes to be joined, wiring the rubber tube used as a connector firmly into place, and surrounding the tube with a bath of glycerine to preserve the rubber from drying and cracking. This arrangement showed upon careful tests no indication of leakage either of air or of moisture; and it is believed that such could not have occurred in amounts sufficient to vitiate the results to any appreciable extent, in view of the other precautions taken against accumulated moisture. Leakage into the space whose pressure was  $p_3$ , unless absurdly large in amount, could not cause error.

4. More or less complete clogging of the capillary tubes which would bring about any change in diameter was not indicated either by the course of the direct experiment, or by the check measurement of  $K$  made at several times.

5. Badly shaped connecting tubes would have introduced frictional resistance of an uncertain magnitude, but the tubes used were designed of suitable size to make the velocity through them so small as to require a motive pressure certainly less than 0.01 mm.

6. There arises naturally a question as to how far the pressure at which the gases are transpired affects the viscosity, a point which is discussed further at page 18 of this paper. That this effect is not

outside the limits of error of observation in this method is however shown by the groups of measurements of  $K$  at various pressures as given in experiments 26 to 38 on  $\text{CO}_2$ . These show that for pressures  $p_1 - p_2$ , as given in the subjoined table, varying from 159.7 to 54.9 mm., the mean values of  $K$  deduced differ at most by less than 0.14 per cent, and the difference is also not progressive in either direction, but irregular, the means for the extremes of pressure being precisely the same.

$p_1 - p_2$ .	$K$ .
159.7 mm.	1.0387
70.9	1.0373
61.9	1.0375
54.9	1.0386

In addition to this, the close agreement of the results of the first series on  $\text{CO}_2$  with those of the second series afford an important check upon the latter, since the former were made with wholly different measuring and heating apparatus, and at pressures from two to three times as great.

7. It should be here noted, that any unequal action at the entrance of the various tubes which might vitiate results obtained by any direct method are to a large extent eliminated by the differential method here used, and would probably enter, if at all, as errors of the second order of magnitude; also, as elsewhere stated, that the surface friction or slip of the gas over the walls of the tube would influence the results only in so far as it changed with the temperature, and would therefore probably be imperceptible.

*Variable errors* may enter from instrumental sources in readings of the barometer or of the cathetometer. Errors of observation in the measurement of the length of the mercury columns by use of the cathetometer would rarely exceed 0.02 mm.; but this amount is considerably less than the pressure changes which occurred in the average experiment. The barometer reading was also obtained with a deviation of less than 0.02 mm. as far as concerns errors of observation. To eliminate as far as possible progressive changes in  $p_1$ ,  $p_2$ , and  $p_3$  during the limits of time required for making a complete set of observations at any given temperature, a pair or more of readings of each was taken, symmetrically distributed in regard to time, so that the means should correspond to about the same instant, and thus be as nearly as possible equivalent to simultaneous observations. The order of readings in the most complete sets would thus usually be barometer, thermometer giving temperature  $t$  of bath, gauges  $A$ ,  $B$ ,  $C$ , temperature

$t$ , gauges  $C$ ,  $B$ ,  $A$  (reversing order on ends of column also), temperature  $t$ , barometer, and finally the readings of auxiliary thermometers for temperatures of gauges, etc.

In order to eliminate as far as possible any uncertainty in setting the cross-hairs of the telescope upon the true top of the mercury meniscus, a difficulty requiring special precaution when readings to 0.01 mm. are attempted, the device was adopted of placing about the tube a clasp with a dull black internal surface. This clasp was placed upon the tube above the mercury column, and was pushed down the tube until the plane of its lower edge was nearly tangent to the mercury meniscus, which then appeared black against a bright background otherwise secured for it. Another convenient method is to make the meniscus bright against a dark background, which is accomplished readily by placing behind the tube a piece of paper separated along a horizontal line into two portions, the upper of which is white, the lower black. When a reading is to be taken, this is adjusted at such a height that the black shall be just visible above the top of the meniscus. In both of these methods it is essential that care be taken that the adjusted black edge be not more than about one fifth of a millimeter above the top of the meniscus, as otherwise error may be introduced through mistaking the reflection of the line by the meniscus for the actual top of the meniscus.

Errors in observation of the height of the barometer affect  $p_1$ ,  $p_2$ , and  $p_3$  by equal amounts in any given experiment. The effect of such errors will therefore be small, and its magnitude, given in the last column of the preceding table, may be demonstrated in the following manner. In the expression

$$y = K \cdot \frac{p_2^2 - p_3^2}{p_1^2 - p_2^2} \cdot \psi(t)$$

may be inserted  $p_1 = H + c$ ,  $p_2 = H + b$ ,  $p_3 = H + a$ , where  $H$  is the barometric reading and  $c$ ,  $b$ , and  $a$  are the gauge readings, at entrance, middle, and exit of the apparatus. Thus,

$$y = K \cdot \frac{b-a}{c-b} \left( 1 + \frac{a-c}{2H+c+b} \right)$$

$$\therefore \frac{dy}{dH} = -K \cdot \psi(t) \cdot \frac{a-b}{b-c} \cdot \frac{2(a-c)}{(2H+b+c)^2}$$

$$\therefore \Delta H = -\frac{c-b}{b-a} \cdot \frac{(2H+b+c)^2}{2(a-c)} \cdot \frac{1}{K \cdot \psi(t)} \cdot \Delta y.$$



As shown by the number deduced from this expression and given in the table, the value of  $\Delta H$  for  $\Delta y = 0.1$  per cent will be at least 2 mm. throughout the range of these measurements. Also when it is considered that constant barometric errors, since they enter with nearly the same magnitude and effect, but with opposite signs, in the determinations of  $K$  and of  $y$ , become inappreciable in effect when not amounting to several millimeters. That the value of  $\Delta H$  is so large will be seen, on inspection of the foregoing differential, to be due in part to the small value of the differences of initial and final pressures used in these experiments.

An inspection of measurements 26 to 38 of Table VI. will show that the average deviation in these determinations of  $K$  is 0.2 per cent. The average deviation in  $K$  which could come from the above-mentioned sources will be seen, however, to be less than 0.2, probably as small as 0.1 per cent, since  $K$  as measured is a function of  $p_1$ ,  $p_2$ , and  $p_3$  only. It thus appears probable that the assumed precision in the instrumental measurement of the pressures is not far in error.

Among other sources of variable error are :—

1. Irregular action of the aspirator causing more or less irregular changes in the amount of pressure at the exit of the gas, i. e. corresponding to  $p_3$ . Fluctuations of this sort would of course be felt at once on the gauge, but the corresponding adjustment of  $p_2$  would require some time, as it would require the transpiration of a definite amount of gas from the  $p_2$  space to the  $p_3$  space. By the use of the regulator which I have devised for the purpose, and which has been already described at page 5, it has been found feasible to keep the pressure constant within a millimeter during a whole day's experimenting, while the water pressure would run from 20 to 35 pounds at frequent intervals. And, as will be seen from an inspection of the values of  $p_3$  given in the tables, the pressure during a series of half a dozen experiments frequently remained constant within 0.2 or 0.3 mm. In a single experiment a change of 0.2 mm. was rare, the average being under 0.1 mm., including errors of reading by the cathetometer. It is thus evident, I think, that this irregularity in  $p_3$ , which is of the nature of an accidental error, since it may be of any magnitude and of either sign, would on the average introduce into  $y$  or  $K$  a deviation not exceeding 0.2 per cent, and probably less than that, and is therefore but little greater in its effect than other sources of accidental error already mentioned.

2. Owing to the want of perfect constancy in the temperature of the oil-bath, and to a slight opportunity for irregularity in the heating

of the conducting tubes at the point of entrance to the bath, the pressure  $p_2$  as measured at any instant might not correspond precisely with the temperature,  $t$ , observed at the same instant, since time would be required for the pressure to adjust itself by an increase or decrease of transpiration. This would, of course, slightly vitiate the result, and the amount of the effect would naturally increase with the temperature. Being a variable error, it would be partially eliminated by multiplied observations. An inspection of the records of readings giving  $p_2$  shows that the average deviation of the pairs of readings taken in the same experiment at any given temperature increases nearly as the temperature from 0.03 mm. at 20° to about 0.1 mm. at 200° C. This in connection with the column headed  $\Delta p_2$  in Table VIII. demonstrates that these observed variations in  $p_2$  were capable of producing average variable errors in  $y$  of from 0.1 per cent to 0.5 per cent, according to the temperature.

If the average deviation of the various groups of determinations of  $\eta_t : \eta_0$  in Table VI. be compared with the corresponding numbers in the table of errors, it will be seen that observed percentage deviations increase with the temperatures, while the precision required in the instrumental measurements increases much less rapidly in any case, in some cases not at all. Also the observed deviations are considerably greater than can be due to instrumental errors, including the thermometer. It seems to be thus indicated that this unequal or irregular heating is probably a correct explanation of the observed variations in  $p_2$ , and is one of the most prominent sources of error in this investigation, though at the same time one which might readily be removed by modifications of the apparatus which should lead to the maintenance of more constant temperatures of the bath, and to the elimination of any irregular heating of parts of the tube.

3. Errors in the determination of the temperatures  $t$  of the gauges would affect the values of  $p_1$ ,  $p_2$ , and  $p_3$ , through the reduction to 0°. Calling 0.00018 the coefficient of absolute expansion of mercury (the cathetometer-bar expansion was separately corrected), the correction to any observed gauge reading,  $l$ , would be, of course,  $C = 0.00018 \, l \, t$ , whence

$$\frac{dc}{dt} = 0.00018 \, l, \quad \text{and} \quad \Delta t = \frac{\Delta c}{0.00018 \, l}.$$

Assuming  $\Delta c = 0.01$  mm., and  $l = 100$  mm.,

$$\Delta t = \frac{0.01}{0.00018 \times 100} = \frac{0.01}{0.018} = 0.55,$$

from which it is obvious that, as the longest column of mercury read in the gauges was about 100 mm., no appreciable error would be introduced into any of these columns through errors less than  $1^\circ$  in their temperature measurements. With the arrangement of screens used, and taking the temperature by means of a thermometer dipping into mercury in a tube of similar dimensions and similarly exposed to the gauge tubes, no error of this magnitude could enter into  $t$ . It is also obvious that, as the gauges were placed side by side and under similar conditions in every respect, the same proportionate error would be produced in each, and this would therefore be largely eliminated. Also, for the reason given in considering the effect of barometric errors, the effect of errors in the determination of its temperature would be wholly imperceptible, and indeed all temperature correction both of the gauges and of the barometer might be omitted without seriously affecting the accuracy of the results, for the reasons already shown, and because the temperatures of the mercury columns differed but a few degrees throughout the whole research.

## II. $\Delta K$ .

*Constant errors* in the determination of  $K$  may arise from any of the sources of error in measurement of  $p_1$ ,  $p_2$ , and  $p_3$ , just discussed, which can occur under the conditions which hold during the determination of  $K$ . These conditions have already been described in detail, and it has been shown that

$$K = \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2},$$

when both capillaries are at the same temperature, conveniently that of melting ice. By the method used in a previous paragraph on variable barometric errors, it may be shown that a constant error even as great as 1 cm. would have an imperceptible effect on  $K$ , and this effect would be counterbalanced and eliminated by a similar and opposite effect of this error in measurements taken with the same barometer to determine  $y$ . To a less extent the same is true of constant errors of the cathetometer scale, and it may be asserted that both of these sources of error are practically eliminated from the results of this investigation. Further sources of constant error have been sufficiently discussed in the paragraphs I. 1-7, and it should be added that, in the packing of ice about the tubes, particular care was used, not only to obtain thorough contact and surrounding at first, but to see that this was maintained.

The *variable errors* attending the determination of  $K$  are those affecting  $p_1$ ,  $p_2$ , and  $p_3$ , and already discussed; and from the measurements of Table VI. just referred to, it appears that the variable errors arise about equally from instrumental measurements and from fluctuations in  $p_3$ .

### III. $\Delta A$ .

As shown by the column headed  $\Delta A$  in the tables, the accuracy necessary in the value of  $A$ , the coefficient of linear expansion of glass used in the computations, increases as the temperature rises, an error of 0.1 per cent in  $y$  corresponding, however, to an error of 20 per cent in  $A$  at the highest temperature attained. The value of  $3A$  used was as follows:—

At	$3A$ .	At	$3A$ .
0°	0.0000245	120°	0.0000266
40°	251	160°	275
80°	258	200°	284

These values were selected as corresponding quite closely to the probable change of the coefficient with change of temperature. The absolute value from 0° to 100° is based on measurements made on the same tube used, but the variation with the temperature was not measured. The error in this coefficient at 100° cannot, I think, exceed five per cent, and is not likely to reach that amount; and a consideration of the results obtained by many observers, besides those which I have myself made on similar kinds of glass, renders it certain that the value of  $3A$  at 200° cannot be much more in error than at 100°. Even had a constant coefficient been employed throughout, its error introduced into  $A$  could hardly have exceeded 15 per cent at any temperature, and had this coefficient been arbitrarily assumed from the results of other observers, an error exceeding 20 per cent would have been unlikely. Since the value of  $\Delta A$  in Table VIII. is 19 per cent at 225°, it appears that, with the values used, the results of the computations of  $y$  cannot have been sensibly in error from this source, even at the highest temperatures attained.

### IV. $\Delta a$ .

The most important quantity whose value is necessarily assumed in the computation of  $y$  is the coefficient of expansion of the gas used. This quantity and the temperature of transpiration, as may be seen from the columns headed  $\Delta a$  and  $\Delta t$ , or from the expression giving  $y$ ,

affect the value of  $y$  to an equal extent, and the two must be determined with a precision ranging from 0.57 per cent at  $60^\circ$  to 0.22 per cent at  $225^\circ$ , in order that each shall produce in  $y$  an error of less than 0.1 per cent. It is well known that the value of  $\alpha$  for all gases changes with the pressure and temperature under which they expand. In the case in hand, the gas expands under the constant pressure  $p_2$  from the temperature  $0^\circ$  of the first capillary, to that  $t$  of the second. The experiments of Regnault furnish the best data for this coefficient, and I have adopted his figures without reducing them for the corrected value of the expansion of mercury, as the small change which would be necessary would not sensibly affect my results. Regnault gives as the coefficient of expansion of carbonic acid under constant pressure

At 760 mm.	$100 \alpha = 0.37099$
“ 2520 “	“ $= 0.38455$

Although the function connecting the pressure and coefficient of expansion is unknown, yet simple extrapolation from these data gives values of  $\alpha$  of sufficient relative precision for the present results, as the total variation of  $\alpha$  within the range of  $p_2$  used is only about 0.12 per cent. The values actually used are given in the tables of experimental data, so that the general effect of this correction can there be noticed, and the data are sufficiently complete to allow a recalculation with other values if desirable. It will be seen here also, as in the value of  $A$ , the change in the coefficient is so small as to produce effects of less than 0.1 per cent if neglected altogether, but the retention of both favors the elimination of accumulated systematic error in  $y$  as a function of  $t$ . For this reason, a similar precaution has been taken in the computations for air, although in that case it is even less necessary.

#### V. $\Delta t$ .

The precision necessary in the measurement of  $t$ , the temperature of the second capillary, is shown in the column headed  $\Delta t$  in Table VIII.

1. As this investigation was aiming at the determination of the change of rate of variation of the viscosity with the temperature, and as this change was known to be small, it seemed of special importance that all precautions should be taken to eliminate systematic errors. And as thermometric measurements, even when conducted with more than ordinary care, are particularly liable to such errors, especial atten-

tion was devoted to this branch of the work. The precision obtained in the thermometry was doubtless greater than was necessary to correspond to that obtained in other parts of the determination of  $y$ , especially below  $60^{\circ}$ ; but it is doubtful if a satisfactory elimination of constant errors could have been obtained with any less care.

The investigations on thermometry which I have made, and still have under way, will be discussed here only in their relation to the measurements given in this paper. The important problem of how far the mercurial thermometer is available as a convenient representative (proxy) for the air thermometer at temperatures up to  $300^{\circ}$  C., and under what conditions and methods it shall be so used, does not enter to any considerable extent into the work in hand, on account of the comparatively small degree of accuracy required in my measurements, even at the highest temperature ( $225^{\circ}$ ) used. The results which I have already obtained in this direction I hope to supplement by enough other material of the same nature to render the whole of sufficient value to be given by itself at some future time.

All the thermometric readings were taken with the bulb immersed in the oil-bath around the second capillary to a depth of about two inches. Above this point, the projecting stem was surrounded by a thin-walled glass tube, closed at the bottom by a rubber stopper through which the thermometer stem passed, and filled with water or glycerine, according to the temperature of the oil. This stem bath was stirred by a vertically moving stirrer, and its temperature was determined at one or more points, according to the length of the mercury column enclosed, by one or more auxiliary thermometers.

The readings were properly corrected for calibration error (except in the case of the Baudin normals, which were found by calibration to be accurate to two or three tenths of a division), for temperature of exposed stem, for true value of unit of scale (error of fixed points), and for deviation from the air thermometer. Poggendorff's correction becomes unnecessary where the mercurial thermometer, as in this work, is compared with the air thermometer direct, and all readings are reduced to that standard.

The thermometers used were respectively, one by Casella of London, No. 32378, and two by Baudin of Paris, Nos. 7335 and 7789. The following table gives the description of these, and of several others to be referred to later.

TABLE IX.

Mark.	Approx. Length in cm.	Range.	Gradua- tion.	Length occupied by 1° C.	Maker.	When made.	Owner.
7335	46	$\overset{\circ}{100}$ to $\overset{\circ}{200}$	$\overset{\circ}{0.1}$ C.	cm. 0.5	Baudin.	October, 1878.	S. W. Holman.
7789	40	100 to 200	0.2 C.	.....	Baudin.	.....	S. W. Holman.
32378	46	-15 to 110	0.2 C.	0.32	Casella.	{ Kew Register, 1873, -4, -8.	{ S. W. Holman.
6163	50	-6 to 40	mm.	0.9	Baudin.	1876-77.	{ Physical Labora- tory, Johns Hop- kins University.
7334	61	0 to 100	$\overset{\circ}{0.1}$ C.	0.5	Baudin.	October, 1878.	{ S. W. Holman.
Kew 104	66	-21 to 103	0.5 F.	0.46	Welch.	July, 1853.	{ Professor Barker, University of Pennsylvania.
Geissler	45	-3 to 102	0.1 C.	0.38	Geissler.	(?)	{ Chemical Labora- tory, Johns Hop- kins University.
368	48 {	About 0 to 100	Arbitrary equal volumes.	} 0.32	Fastré.	1851.	{ Professor Gibbs, Harvard College.
376	48 {	About 0 to 100	Arbitrary equal volumes.		Fastré.	1851.	{ Professor Gibbs, Harvard College.
3235	40	32 to 212	$1^{\circ}.0$ F.	0.16	Casella.	(Quite old.)	{ Harvard College Observatory.

It is always somewhat unfortunate, but usually unavoidable, to have the stem of the thermometer at a temperature lower than that of the bulb, and the correction from this cause increases about as the square of the temperature of the bulb. Among the discussions of the methods of correction for this error may be cited those of Kopp, Regnault, Holtzman, and, more recently, Wüllner\* and Mills.† After a careful experimental trial and consideration of these, it appeared to me that, for the present purposes, results fully equal to the requirements would be obtained by the use of a stem bath as above described.

Let  $s$  = mean temperature of this bath.

$t$  = actual temperature of bulb (to be determined).

0.000156 = coefficient of apparent expansion of mercury in glass.

$n$  = length of mercury column in units of the scale graduation at temperature  $s$ .

$r$  = direct reading of thermometer corrected for calibration error.

Then

$$t = r + 0.000156 (t - s)n,$$

or approximately

$$t = r + 0.000156 (r - s)n,$$

\* Lehrbuch der Experimental-Physik, iii. 312.

† Proc. Royal Soc. Edinburgh, xxix. 569.

since  $t = r$  very nearly. When, however,  $t - r$  becomes sufficiently large, the value of  $t$  as computed by the second expression must be considered as a first approximation only, and a second or third must be computed.

Without entering into the further discussion of thermometry which I have made, a portion of which was presented in a paper (unpublished) read at the Boston Meeting of the American Association for the Advancement of Science in 1880, I will assert that the errors in my thermometric work are below the values assigned to  $\Delta t$  in Table VIII.

In order to determine the deviation of the mercurial thermometers used in my measurements from the air thermometer, a direct comparison was made of these and several others, the results of which possess some interest, and will be here given.

Through the kindness of Prof. H. A. Rowland, to whom I would here express my sincere thanks for his courtesy and assistance, and by an appropriation granted by the Rumford Committee of the American Academy, I was enabled to visit Baltimore in January, 1879, and to make, in co-operation with Prof. Rowland, some thermometric comparisons, using an apparatus designed by him.

The method and apparatus for comparisons at temperatures below  $100^{\circ}$  C. were those described and discussed in full by Prof. Rowland in his valuable memoir on the Mechanical Equivalent of Heat.\* It is unnecessary to reproduce the description here further than to say that a water bath was used in the apparatus, that the whole thermometer was immersed, and that in the discussion of the accuracy of the apparatus Prof. Rowland writes, † “From this table it would seem that there should be no difficulty in determining the  $40^{\circ}$  point on the air thermometer to at least 1 in 2000; and experience has justified this result.” It is quite possible, however, that the result obtained in my comparison was not of quite as high precision as this, but it did not fall far short.

On two successive days a group of four thermometers was compared with the air thermometer, with the results shown in the two following tables. Thermometer Baudin 6163, whose error had already been carefully studied by Prof. Rowland, was included in both series. Table IX., on page 38, gives the complete description of the various instruments. Casella 32378 was used in my measurements below  $100^{\circ}$ . It is a fairly good instrument, but decidedly inferior to any of

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\* These Proceedings, June, 1879, vol. xv. pp. 77 *et seq.*

† Ibid., p. 99.



the Baudin thermometers, both from its quite considerable calibration error, which is irregular between the  $95^{\circ}$  and  $100^{\circ}$  points, and from the lack of the calibration chamber at the top, as well as from the general form of the tube and bulb. The Kew register was also obtained, but was not used, as it was given only to  $0^{\circ}.1$ , i. e. half-divisions of the scale, and corresponds neither with the result of the comparison between Kew Standard No. 104, as given below, with the ascertained calibration error, nor with the air thermometer error. The pair of Fastré tubes, kindly lent for the comparison by Dr. Wolcott Gibbs of Harvard University, were of special interest, as being probably nearly identical with the crystal glass used by Regnault and calibrated by the method employed by him, Fastré being the maker of many of Regnault's thermometers.

The chief special points of interest in the results of these comparisons are indicated by Prof. Rowland at the place cited, and I will allude here only to the fact that all of the mercurial thermometers used in this study appear to read higher than the air thermometer below  $100^{\circ}$ .

TABLE X.

Air Thermometer.	Original Readings.					Reduced Readings.				
	6163 Baudin.	7334 Baudin.	Kew Standard No. 104.	32378 Casella.	Geissler.	6163 Reduced to Air Thermometer.	7334 Baudin.	Kew Standard No. 104	32378 Casella.	Geissler.
0.	*58.83	-0.11	32.68	+0.20	+0.69	0.	0.	0.	0.	0.
10.43	63.5	....	33.60	0.71	....	0.52	....	0.52	0.51	....
6.08	113.0	....	43.65	6.33	....	6.08	....	6.11	6.13	....
12.68	171.55	12.59	55.47	12.91	13.42	12.65	12.73	12.68	12.70	12.82
20.49	242.0	20.48	69.55	20.77	21.29	20.49	20.63	20.57	20.56	20.74
24.55	278.8	24.50	76.90	24.80	25.33	24.54	24.66	24.61	24.59	24.81
29.51	323.9	29.49	85.88	29.80	30.32	29.52	29.66	29.61	29.58	29.83
39.45	413.1	39.43	103.72	39.76	40.22	39.47	39.62	39.53	39.54	39.80
39.15	410.7	39.15	103.23	39.48	39.98	39.20	39.34	39.26	39.26	39.56
51.17	....	51.10	124.84	51.49	51.83	....	51.32	51.29	51.26	51.49
61.12	....	61.05	142.73	61.47	61.69	....	61.29	61.24	61.23	61.41
70.74	....	70.57	159.87	71.00	71.14	....	70.83	70.78	70.76	70.92
80.09	....	79.74	176.50	80.31	80.25	....	80.02	80.04	80.06	80.10
80.89	....	80.15	177.23	80.74	80.66	....	80.43	80.44	80.49	80.51
89.95	....	89.63	194.35	90.22	90.11	....	89.93	89.97	89.97	90.03
89.92	....	89.59	194.22	90.18	90.06	....	89.89	89.90	89.93	89.98
100.00	....	99.69	212.37	100.06	99.32	....	100.00	100.00	100.00	100.00

\* The original readings in ice were 58.68 and 58.45, to which .15 was added to allow for pressure of water in the comparator. This, of course, gives the same final result as if .15 were subtracted from each of the other temperatures. No correction was made to the others.

† Probably some error of reading.

TABLE XI.

Air Ther- mometer.	Original Readings.					Reduced Readings.				
	6163 Baudin.	376 Fastré.	7316 Baudin.	368 Fastré.	3235 Casella.	6163 Reduced to Air Ther- mometer.	376 Fastré.	7316 Baudin.	368 Fastré.	3235 Casella.
0.	*58.60	111.3	-0.23	87.6	32.80	0.	0.	0.	0.	0.
3.67	90.7	130.0	....	106.25	39.35	3.61	3.64	....	3.64	3.65
11.55	161.6	170.9	11.40	147.2	53.70	11.56	11.60	11.64	11.62	11.63
20.72	243.7	217.9	20.59	194.2	70.15	20.70	20.75	20.84	20.80	20.79
32.19	347.4	276.9	32.09	253.2	90.80	32.17	32.24	32.34	32.28	32.29
39.36	411.85	313.85	39.26	290.1	103.68	39.36	39.43	39.52	39.48	39.45
50.71	....	372.0	50.57	348.2	123.65	....	50.75	50.84	50.80	50.57
60.10	....	420.0	59.92	396.45	140.80	....	60.10	60.19	60.21	60.12
73.82	....	490.6	73.59	466.85	165.68	....	73.84	73.87	73.93	73.97
86.50	....	555.25	86.16	531.22	188.20	....	86.48	86.51	86.56	86.56
....	....	550.2	85.21	525.95	186.42	....	85.45	85.50	85.45	85.51
100.00	....	624.93	99.70	600.58	212.45	....	100.00	100.00	100.00	100.00

The comparisons above 100° were of rather more importance than those below for the purpose of the present investigation. For this work the newly constructed apparatus designed by Prof. Rowland was put in use for the first time.

It consists of a massive copper vessel of circular section, having inner dimensions of 18.5 cm. depth and 23 cm. diameter, and with walls and bottom 1.2 cm. thick. Upon the carefully turned flange which forms the upper edge of the walls is bolted a correspondingly heavy circular copper plate, a projection from which enters a short distance into the vessel. From the middle of this plate, and cast upon it, rises a heavy copper bar of 10 cm. by 6 cm. cross-section outside, and 55 cm. high, but containing a vertical channel in its broader front face having a depth of 3.5 cm. and a width of 8 cm. Sheet-iron covers of suitable length and about 4 cm. width are provided to close those portions of the front of this channel which should be closed during the work. The cylindrical vessel up to the flanges is placed within a sheet-iron casing, from which rises at the back a sheet-iron outlet pipe which extends upward along the back of the channelled bar to its top. The heated gases escaping through this flue from the Bunsen burner placed beneath the copper vessel heat the bar to any desired temperature, regulation being had by means of inlet valves admitting cool air at the base of the flue.

\* See note to preceding table.

The bulb of the air thermometer is introduced into the middle of the copper vessel, and is surrounded by oil, which completely fills the vessel. The stem of the thermometer passes through a groove provided in the upper surface of the vessel. The mercurial thermometers are placed against the back of the vertical channel, with their bulbs projecting into the oil-bath to the level of the air thermometer bulb. The unoccupied space in the channel between the stems of the thermometers and the sheet iron front is filled with cotton wool or other similar substance, which may be removed at suitable points to take readings. Auxiliary thermometers are placed with the bulbs at various suitable heights along the stems of the other thermometers, and by means of them the temperature of the vertical bar, and thus of the thermometer stems, is maintained at a known temperature very nearly the same as that of the bath, so that the stem exposure correction becomes small and determinate. The massiveness and correspondingly high conductivity and calorific capacity of the whole vessel render it possible to maintain the temperature nearly uniform and constant during the time necessary for the thermometers to acquire the temperature of the bath and the readings to be taken. The oil-bath is thoroughly stirred by a sheet-iron stirrer moving vertically.

The air thermometer used was an imported instrument of the form described by Jolly.\* The glass of the bulb was supposed to be of the kind used by him, and the coefficient of expansion used in my computations was based on that which he has given, and was assumed, with sufficient accuracy for the present purposes, to be, —

$t^{\circ}$	Mean coeff. from $0^{\circ}$ to $t^{\circ}$ .
100	0.0000278
140	286
180	295
200	300
250	31
300	32

The mercurial columns were read by means of black hard-rubber clasps sliding upon the tubes, the scale being upon the surface of a glass mirror. Readings were taken to tenths of a millimeter. For the temperature reductions of the mercury columns it was assumed that the coefficient of apparent expansion of the glass scale was

$$\Delta = 0.000181 - 0.000009 = 0.000172,$$

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\* Pogg. Ann. Jubelband (1874); Amer. Jour. Sci., vi. 591.

and that the scale was correct at  $0^{\circ}$  C., which was certainly quite nearly true. The value of the coefficient of expansion of air found with this apparatus was  $\alpha = 0.003679$ .

In February, 1879, I made a comparison of four thermometers, including Baudin 7335 and 6161, and two other thermometers of little value, at temperatures from  $100^{\circ}$  to  $250^{\circ}$  C., with the apparatus just described. The results were fairly good, but not entirely satisfactory. Later in the same year, Dr. E. H. Hall (then a student at Baltimore) made a similar series of comparisons, using Baudin thermometers Nos. 6161, 7315, 7325, and 7324. His results with No. 6161 differed somewhat from mine, owing chiefly to the progressive change in zero which attended this thermometer during my own use of it, as well as to something of the same change during his own measurements. The results as a whole agree fairly well, and I have deemed it best to average the corrections found on all the Baudin instruments of both series, and to use this average result as a correction to Baudin 7335, which I employed in my subsequent measurements. This seemed to me the safest course, because of the general agreement of all the Baudin thermometers used, and because more comparisons of 7335 could not be made at that time to eliminate the errors entering into a single series. That the table of errors thus obtained and given below is not all that might be desired is obvious, but I am disposed to believe that at  $200^{\circ}$  it is not in error by an amount exceeding  $0^{\circ}.5$ , and by a less amount at lower temperatures, thus introducing into  $y$  an error not exceeding 0.1 or 0.2 per cent.

TABLE XII.

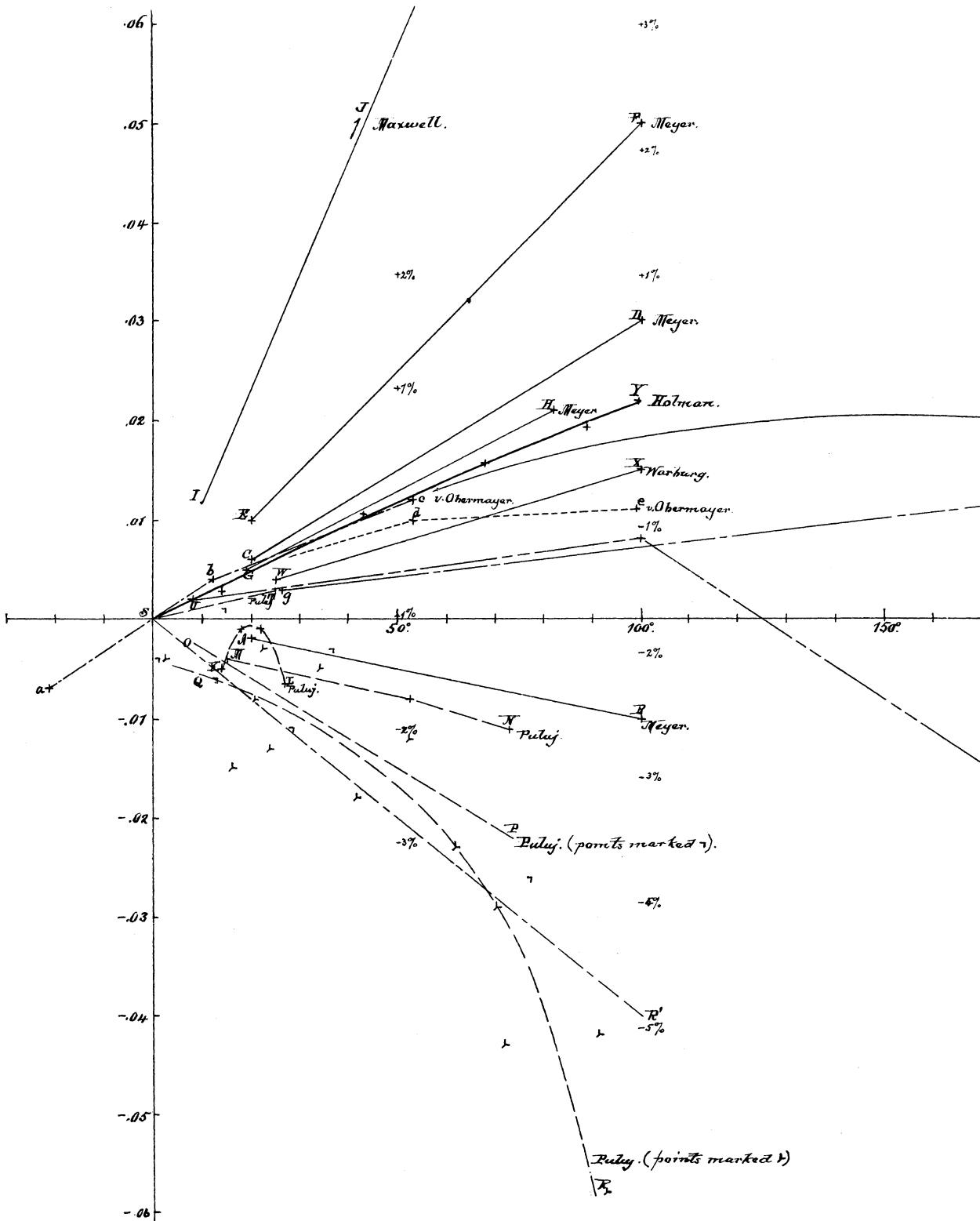
DEVIATION OF BAUDIN No. 7335 FROM AIR THERMOMETER.

$\overset{\Delta}{\underset{\circ}{t}}$	$\overset{\Delta}{\underset{\circ}{}}$	$\overset{\Delta}{\underset{\circ}{t}}$	$\overset{\Delta}{\underset{\circ}{}}$
100	0.0	160	1.45
120	— 0.5	180	1.65
140	— 1.0	200	1.50

The measurements made at  $225^{\circ}$  were taken with Baudin No. 7789, and the correction to be applied to it was ascertained, by a comparison with No. 7335, to be  $+0^{\circ}.8$ .

I have devised and had constructed an apparatus for air-thermometer corrections at high temperatures, and had hoped to be able to make a further series of comparisons of all thermometers used, but for reasons elsewhere stated the work is unfinished, and has no prospect of immediate completion.





13%

P<sub>1</sub> Meyer.

12%

11%

P<sub>2</sub> Meyer.

H<sub>1</sub> Meyer. Y Holman.

X Markburg.

E v. Obermayer.

-1%

100°

-2%

150°

200°

250°

$$\frac{7}{7} = (1 + 0.00367)^{0.76}$$

h<sub>1</sub>  
v. Obermayer.

P<sub>3</sub> Meyer.

-3%

Y  
Niedemann.

h<sub>2</sub>. (points marked 7).

-6%

R'  
-5%

R<sub>2</sub>. (points marked 1)

R<sub>2</sub>

